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SCHOOL OF DISTANCE EDUCATION

ASSIGNMENT QUESTION PAPER 2020 – 21

M.A. / M.Sc. (FINAL) MATHEMATICS

MEASURE THEORY AND FUNCTIONAL ANALYSIS

ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A **(4 x 4 = 16 Marks)**

1. a) State and prove Bounded convergence theorem.

   b) State and prove Monotone Convergence theorem.

2. a) State and prove Reisz Representation theorem.

   b) State and prove Radon-Nikodym theorem.

3. a) Let M be a closed linear subspace of a normed linear space N. if the norm of a coset r+M in the quotient space N/M is defined by

   $$\|x+M\| = \inf\{\|x+m\| : m \in M\}$$

   then show that N/M is a normed linear space. Further, show that if N is a Banach space, then so is N/M.

   b) State and prove the open mapping theorem.

4. a) State and prove Bessel's inequality.

   b) Let H be a Hibert space and Let f be an arbitrary functional in H^*. Then show that there exists a unique vector y in H such that $f(x) = (x, y)$ for each x in H.

SECTION – B **(4 x 1 = 4 Marks)**

5. Answer all the following.

   a) If f and g are bounded measurable functions defined on a set E of finite measure, then show that $\int_E (af+bg) = a\int_E f + b\int_E g$.

   b) Every measurable subset of a positive set is itself positive. Also show the union of a countable collection of positive sets is positive.

   c) If N is a normed linear space and x_0 is a non-zero vector in N, then prove that there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.

   d) If x and y are any two vectors in a Hilbert space, then show that $|(x, y)| \le \|x\|.\|y\|$.

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M.A. / M.Sc. (FINAL) MATHEMATICS

COMPLEX ANALYSIS

ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A **(4 x 4 = 16 Marks)**

1. a) Let z = Cos $\frac{2\pi}{n} + i \sin\frac{2\pi}{n}$ for an integer $n \geq 2$ then show that

   $1 + Z + Z^2 + ... + Z^{n-1} = 0$

   Find Z^n.

   b) Show that a MOBIUS transformation takes circles onto circles

2. a) State and prove Cauchy's integral formula (first version)

   b) Show that $\int_O^\pi \frac{d\theta}{a+\cos\theta} = \frac{\pi}{\sqrt{a^2-1}}$.

3. a) State and prove Cauchy's residue theorem

   b) Sate and prove Arzela – Ascolis theorem

4. a) State and prove Mittag Leffler's theorem

   b) State and prove Schwartzs reflection principle

SECTION – B **(4 x 1 = 4 Marks)**

5. Answer all of the following

   a) Define analytic function and Cauchy – Riemann equations

   b) State (i) Livioulle's theorem and (ii) Morera's theorem

   c) State (i) Montel's theorem and (ii) Runge's theorem

   d) Show that $\int_0^\infty \frac{x^{-C}dx}{1+x} = \frac{\pi}{Sin\pi^C}$ if 0 < C <1

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ASSIGNMENT QUESTION PAPER 2020 – 21

M.A. / M.Sc. (FINAL) MATHEMATICS

NUMBER THEORY

ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A **(4 x 4 = 16 Marks)**

1. a) For $n \geq 1$ prove that $\phi(n) = n \prod_{p/n}\left(1 - \frac{1}{p}\right)$.

   b) If both g and $f * g$ are multiplicative, then prove that f is also multiplicative.

2. a) State and prove Lagrange theorem.

   b) State and prove Chinese remainder theorem.

3. a) Show that there are infinitely many primes of the form $4n + 1$

   b) Show that for $x > 1$ and $x \neq x_1$ we have $L(1, x)\sum_{n \leq x} \frac{\mu(n)x(n)}{n} = 0(1)$

4. a) State and prove quadratic reciprocity law.

   b) State and prove Euler's criterion

SECTION – B

5. Answer all the following. **4 x 1 = 4 Marks**

   a) If $n \geq 1$, then prove that $\log\ n = \sum_{d \leq n} \wedge(d)$.

   b) State and prove Euler Fermat theorem.

   c) Show that there are infinitely many primes of the form $4n$-1.

   d) State and prove Gauss lemma.

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ASSIGNMENT QUESTION PAPER 2020 – 21

M.A. / M.Sc. (FINAL) MATHEMATICS

INTEGRAL EQUATIONS

ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A **(4 x 4 = 16 Marks)**

1. a) Transform the initial value problem $\frac{d^2y}{dx^2}+y=\cos x, y(0)=0, y^1(0)=1$ into an integral equation.

   b) Let $K:[a,b]\times[a,b]\to\Re$ be continuous. Then prove that the only possible continuous solution of $\varphi(x)=\lambda\int_a^x K(x,y)\varphi(y)dy$ is the trival zero solution.

2. a) Prove that if $\varphi(x)=\lambda\int_a^b k(x,y)dy$ has eigen values, where k is a Hermintioan kernel, then they are all real.

   b) State and prove Hilbert Schmidt theorem.

3. a) Solve the integral equation

   $$\int_0^x \sin\alpha(x-y)\varphi(y)dy=1-\cos(\beta x)$$

   where α and β are given constants, by applying Laplace transformations.

   b) Solve the integral equation $\varphi(x)=\int_0^x \frac{1+\varphi(y)}{1+y}dy$ by using Picard's method.

4. a) Write the Picard iterative method for the existence of solution of the nonlinear Volterra integral equation of the second kind.

   $$\varphi(x)=f(x)+\lambda\int_0^x F(x,y,\varphi(y))dy\cdot$$

   b) Find first and second approximations in the iterative solution of the integral equation

   $$\int_0^1 (x+y)^{\frac{1}{2}}[\varphi(y)]^{\frac{1}{2}}dy=\varphi(x)\cdot$$

   and find bounds of $\varphi(x)$.

SECTION – B **(4 x 1 = 4 Marks)**

5. Answer all the following.
   a) Describe the shop stocking problem.
   b) Find the eigen values and eigen functions of the system.

   $$\varphi(x)=\lambda\int_0^1(1+xt)\varphi(t)dt, \qquad 0\le x\le 1.$$

   c) Solve the integral equation .

   $$x^2=\int_o^x \sin(\alpha(x-y))\varphi(y)\,dy,\quad \alpha\neq 0.$$

   d) Find the first three functions in the sequence of functions arising from the iterative solution of the integral equation.

   $$\varphi(x)=x+\lambda\int_0^x[1+x\{\varphi(y)\}]^2\,dy\cdot$$

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ASSIGNMENT QUESTION PAPER 2020 – 21
M.A. / M.Sc. (FINAL) MATHEMATICS
LINEAR PROGRAMMING AND GAME THEORY
ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A **(4 X 4 = 16 Marks)**

1. Define feasible solution, optimal solution of a standard maximization problem and state and prove optimality criterion for standard maxization problem.

2. Solve the following by simplex method
Maximize $2\xi_1 + 4\xi_2 + \xi_3 + \xi_4$
Subject to
$$\xi_1 + 3\xi_2 + \xi_4 \le 4$$
$$2\xi_1 + \xi_2 \le 3$$
$$\xi_2 + 4\xi_3 + \xi_4 \le 3$$
$$\xi_1 \ge 0 \forall i = 1,2,3,4$$

3. Solve the transportation problem whose cost matrices is given as:

	M_1	M_2	M_3	M_4	Supply
P_1	4	4	9	3	3
P_2	3	5	8	8	5
P_3	2	6	5	7	7
Demand	2	5	4	4	

4. Solve the game with pay off matrix
$$\begin{bmatrix} -5 & 5 & 0 & -1 & 8 \\ 8 & -4 & -1 & 6 & -5 \end{bmatrix}$$

SECTION – B **4 x 1 = 4 Marks**

5. Answer all the following.
a) Define (i) Standard maximum problem
(ii) Canouical maximum problem
b) Write the dual of the problem
Maximize $3x_1 + 4x_2 - 2x_3$
Subject to
$$x_1 - 2x_2 + x_3 = 2$$
$$x_1 + x_2 - 3x_3 \le 4$$
$$3x_1 + x_2 + x_3 \ge 5$$
$$-2x_1 - x_2 - x_3 = -3$$
$x_1, x_2, x_3 \ge 0,\ x_2$ is unrestricted and show that dual of dual is primal.

c) Define the terms
(i) Flow
(ii) Maximal flow
(iii) Cut IV minimal cut

d) Define
(i) Mixed strategy
(ii) Pure strategy
(iii) Extended game of a game $\Gamma = (S, T, \phi)$

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ASSIGNMENT QUESTION PAPER 2020 – 21

M.A. / M.Sc. (FINAL) MATHEMATICS

LATTICE THEORY

ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A **(4 x 4 = 16 Marks)**

1. a. Show that every finite partly-ordered set can be represented by a diagram.
   b. Show that in a Lattice L the prescription $a \leq b \Leftrightarrow a \cap b = a \quad (a, b \in L)$ defines an ordering relation.

2. a. Show that every element of a compactly generated lattice can be represented as a meet of a completely meet irreduciable elements.

   b. Show that the dual, every sublattice and every homomorphic image of a modular lattice is modular.

3. a. Show that every complete Boolean algebra is infinitely distributive.

   b. Show that the algebra of relations R(M) of any set M forms a lattice-ordered semigroup with respect to the operations defined in the supplement of the foregoing theorem and to the multiplication of relations.

4. Show that every lattice is isomorphic to some sublattice of a complete lattice; moreever, every modular (distributive) lattice is isomorphic to some sublattice of a suitably chosen-modular (distributive) complete lattice.

SECTION B **4 x 1 = 4 Marks**

5. Answer all the following.:
   a. Write about Galois connection corresponding to the relation ϕ.

   b. write the statements of

      i. Isomorphism theorem of modular lattices

      ii. Kurosh-Ore theorem

   c. Write about Boolean algebras and Boolean rings.

   d. Write about congruence relation of lattices.

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ASSIGNMENT QUESTION PAPER 2020 – 21

M.A. / M.Sc. (FINAL) MATHEMATICS

COMMUTATIVE ALGEBRA

ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A **(4 x 4 = 16 Marks)**

1. a) i) If L $\supseteq$ M $\supseteq$ N are A- modules, then prove that (L/N) (M/N) $\cong$ L/M.
   ii) If (M_1, M_2) are submodules of an A – module M, then prove that
   $(M_1 + M_2)/M_1 \cong M_2/(M_1 \cap M_2)$

   b) State and prove Nakayama's lemma.

2. For an A-module *N*, prove that the following are equivalent:

   a) N is flat

   b) If $0 \rightarrow M' \rightarrow M \rightarrow M'' \rightarrow 0$ is any exact sequence of A – module, then the tensored sequence $0 \rightarrow M' \otimes N \rightarrow M \otimes N \rightarrow M'' \rightarrow 0$ is extract.

   c) If $f : M' \rightarrow M$ is injective, then $f \otimes I \rightarrow M \otimes N \rightarrow M \otimes N$ is injective.

   d) If $f : M' \rightarrow M$ is injective and *M, M'* are finitely generated, then $f \otimes I : M \otimes N \rightarrow M \otimes N$ is injective.

3. a) State and prove second uniqueness theorem.

   b) If r(I) is maximal, then prove that I is primary, In particular, the powers of a maximal ideal M more *M*-primary.

4. a) Prove that the length *l(M)* is an additive function on the class of all A-modules of finite length.

   b) Prove that in a Noetherian ring every irreducible ideal is primary.

SECTION – B

5. Answer all the following **4 x 1 = 4 Marks**

   a) Let M be a finitely generated A – module N a submodule of *M*, $I \subseteq A$ an ideal. Then prove that $M = IM + N \Rightarrow M = N$.

   b) Show that S^{-1} A is a flat A-module.

   c) If $I = r(I)$, then prove that I has no embedded prime ideals.

   d) Prove that in a Neotherian ring A, every ideal a contains a power of its radical.

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ASSIGNMENT QUESTION PAPER 2020 – 21

M.A. / M.Sc. (FINAL) MATHEMATICS

NUMERICAL ANALYSIS AND COMPUTER TEQCHNIQUES

ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A **(4 x 4 = 16 Marks)**

1. Write assignment statements to computer the mass M of an electron using the following formula:

   $V_c = MC^2 - M_0C^2$

   Where

   V = Voltage in the electron gun

   e = Charge of an electron = 1.60 + 10^{-18} Coulomb

   m_0 = rest mass of an electron = 9.11 + 10^{-31} kg

   C = Speed of light 3.00 × 10^8 m/s

2. Evaluate the integral $I = \int_{-1}^{1} \left(1 - x^2\right)^{3/2} \cos x \, dx$ using Gauss-Legendre three point formula.

3. Solve the initial value problem $U^1 = -2tU^2$, $U(0) = 1$ with h = 0.2 on the interval [0,1]. Use the fourth order Classical Runge Kutta method.

4. Write a FORTRAN program to evaluate the integral $\int_1^2 \sin 2x \, dx$ using Simpson's rule with 4 sub intervals.

SECTION – B

5. Answer all the following **4 x 1 = 4 Marks**

   a) Write about the variable with examples.

   b) Determine a, b and c such that the formula

   $\int_0^h f(x)dx = h\left\{af(0) + bf\left(\frac{h}{3}\right) + cf(h)\right\}$ is exact for polynomials of as high order as possible and determine the order of the truncation error.

   c) Given the initial value problem $u^1 = t^2 + u^2, u(0) = 0$ determine the fist three non-zero terms in the Taylor series for *u(t)*

   d) Explain shooting method

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ASSIGNMENT QUESTION PAPER 2020 – 21

M.A. / M.Sc. (FINAL) MATHEMATICS

UNIVERSAL ALGEBRA

ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A **(4 x 4 = 16 Marks)**

1. a. Let P be a poset such that $\wedge A$ exists for every subset A of P or such that $\vee A$ exists for every subset of P A. Then prove that P is a complete lattice.

   b. Prove that every algebraic lattice is isomorphic to the lattice of closed subsets of some set *A* with algebraic closure operator *C.*

2. a. If *A* is congruence permutable, show that *A* is congruence modular.

   b. State and prove third isomorphism theorem.

3. a. With usual notation, prove that $V = HSP$.

   b. Suppose $U_1(X_1)$ and $U_2(X_2)$ are two algebras in a class *k* with the universal mapping property for *k* over the indicated sets. If $|X_1| = |X_2|$ then prove that $U_1(X_1) = U_2 X_2$.

4. a. Show that if *L* is a sub directly irreducible distributive lattice then $|L| \le 2$.

   b. State and prove Stone Duality theorem.

SECTION – B

5. Answer all the following.: **4 x 1 = 4 Marks**

   a. Define a modular lattice. Show that every distributive is a modular lattice.

   b. State and prove second isomorphism theorem.

   c. Define a sub direct product, a sub direct embedding and subdirectly irreducible.

   d. Show that in a distributive lattice relative complements are unique if they exist.