ANDHRA UNIVERSITY

SCHOOL OF DISTANCE EDUCATION

ASSIGNMENT QUESTION PAPER 2020 – 21

M.A. / M.Sc. (PREVIOUS) MATHEMATICS

ALGEBRA

ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A (4 x 4 = 16 Marks)

1. a) Show that a homomorphism $\phi : G \rightarrow H$ from a group G to group H is injective if and only if ker $\phi = \{e\}$.

   b) Let H and K be subgroups of a group $(G, .)$. Then show that the product HK is a subgroup of G if and only if $HK = KH$.

2. a) Show that alternating group A_n is generated by the set of all 3-cycles in S_n.

   b) Show that the alternating group A_n is simple if simple if n > 4.

3. a) Show that in a non zero commutative ring which unity an ideal M is maximal if and only if R / M is field.

   b) If R is a commutative ring then show that an ideal P in R is prime if and only if ab $\in$ P, $a \in R, b \in P$ implies $a \in P$ or $b \in P$.

4. a) State and prove Eisentein criterion

   b) Let $P(x)$ be an irreducible polynomial in $F[x]$ then show that there exists an extension E of F in which $P(x)$ has a root.

SECTION – B

5. Answer all the following. **4 x 1 = 4 Marks**

   a) Show that center of a group is a normal subgroup of G.

   b) Find the non isomorphic abelian groups of order 720.

   c) Show that every Euclidean domain is a PID.

   d) Let $f(x) \in F(x)$ be a polynomial of degree 2 or 3 then show that $f(x)$ is reducible if and only if $f(x)$ has a root in F.

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M.A. / M.Sc. (PREVIOUS) MATHEMATICS

LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A **(4 x 4 = 16 Marks)**

1. a) Prove that every n-dimensional vector space over the field F is isomorphic to the space F^n.

   b) Let V be a finite dimensional vector space over the field F and T be a linear operator on V. Prove that T is diagonalizable it and only if the minimal polynomial for T has the form $P(x) = (x-c_1) \ldots.(x - c_k)$ where $c_1, c_2, \ldots.. c_k$ are distinct elements of F

2. a) Let $y_1(x)$ and $y_2(x)$ be two solutions of the second order differential equation
   $y^{11} + p(x).y^1 + Q(x).y = 0$ on [a, b].
   The prove that their wronskian
   $W = w^1(y_1,y_2)$ is either identically zero or never zero on [a, b]

   b) Find the general solution of $y^{11} - 2y^1 = 12x - 10$ by the method of undetermined coefficients.

3. a) Solve the system

$$\frac{dx}{df} = 3x - 4y$$

$$\frac{dy}{dt} = x - y.$$

   b) State and prove picard's Theorem.

4. a) Define Laplace transform and solve the following differential equation by applying Laplace transformation method

$$y^{11} + 2y^1 + 2y = 2,$$

$$y(0) = 0, y^1(0) = 1.$$

   b) Solve the integral equation $y(x) = x^3 + \int_0^x \sin(x-t)y(t)dt$ using Laplace transform method.

5. Answer all the following **4 x 1 = 4 marks**

   a) Define similar matrices and prove that similar matrices will have the same characteristic polynominal.

   b) What are the Voltera's pre-predator equations? Describe the dynamic behavior of these equations.

   c) Find the general solution of $2y^{11} + 2y^1 + 3y = 0$.

   d) If $x > 0$ then show formally that $\int_0^\infty \frac{\sin xt}{1+t^2} dt = \pi/2$.

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M.A. / M.Sc. (PREVIOUS) MATHEMATICS

REAL ANALYSIS

ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS

SECTION – A **(4 x 4 = 16 Marks)**

1. a) In a compact metric space, prove that every sequence has a convergent subsequence.

   b) Prove that closed subset of a compact set is compact.

2. a) State and prove necessary and sufficient condition for the existence of Riemann-Steltjes integral of bounded function with respect to a monotonically increasing function.

   b) State and prove fundamental theorem of integral calculus.

3. a) State and prove Cauchy criterion for uniform convergence of sequences of functions.

   b) Let f $\in$ R on $[a,b]$ for $a \le x \le b$, put $F(x) = \int_a^x f(t)dt$. Then prove that F is continuous on $[a,b]$. Further, prove that F is differentiable at x_0 and $f^1(x_0) = f(x_0)$, whenever f is continuous at x_0.

4. State and prove inverse function theorem.

SECTION – B

5. Answer all the following. **(4 x 1 = 4 Marks)**

   a) Prove that every convergent sequence is a Cauchy sequence.

   b) Prove that every differentiable function is continuous.

   c) State and prove integration by parts formula.

   d) State Implicit function theorem.

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ASSIGNMENT QUESTION PAPER 2020 – 21

M.A. / M.Sc. (PREVIOUS) MATHEMATICS

TOPOLOGY

ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A (4 x 4 = 16 Marks)

1) a) Define a metric space. Give an example of a metric space. In a metric space X prove that a subset F of X is closed it and only if F^C is open in X.

   b) Define a continuous function on a metric space. Prove that a function f from a metric space X into a metric space Y is continuous if and only if $x_n \rightarrow x$ in $X \Rightarrow f(x_n) \rightarrow f(x)$ in Y.

2) a) Define (i) Base and (ii) Subbase for a topology on a nonempty set X. (iii) prove that the set of all open intervals in $\Re$ forms a sub base for the standard topology of $\Re$ and also prove that the set of all open rays from a base for the standard topology of $\Re$.

   b) State and prove Heine–Borel theorem

3) a) State and prove Tychonoff's theorem

   b) State and prove Lebesgue covering Gmma

4) a) Prove that product of Hausdorff spaces is Hausdorff.

   b) State and prove Weierstrass approximation theorem

SECTION – B (4 x 1 x 4 Marks)

5) Answer all of the following

   a) State and prove Cantor's intersection theorem.

   b) State and prove Lindelos's theorem.

   c) Prove that continuous image of a connected set is connected.

   d) Prove that every compact metric space is complete and totally bounded.

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ASSIGNMENT QUESTION PAPER 2020 – 21
M.A. / M.Sc. (PREVIOUS) MATHEMATICS
DISCRETE MATHEMATICS
ANSWER ALL QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.

SECTION – A **(4 x 4 = 16 Marks)**

1. a) Prove that there are $\frac{1}{2}(n+1)$ pendant vertices in any binary tree with n vertices.

   b) Prove that a graph G is Eulerian if and only if every vertex of G is of even degree.

2. a) State and Prove De-Morgan's Laws in a Boolean algebra

   b) Prove that the cardinality of a finite Boolean algebra is always of the form 2^n and any two Boolean algebras with the same cardinality are isomorphic.

3. a) Let F and F^1 be free semigroups on B and B^1 respectively. Then prove that $F \simeq F^1$ if and only if $|B| = |B^{'}|$

   b) Let an automator $A^{'}$ be homorphic image of an automator A. Then prove that the monoid $M_{A'}$ is a homorphic image of the monoid M_A.

4. a) Let C = < S> be the linear code generated by a subset S of K^n. Then prove that dimension of (C) + dimension $(C^{\perp})$= n.

   b) Prove that a matrix H is a parity-check matrix for some linear code C if an only if the columns of H are linearly independent.

SECTION – B

5. Answer all the following. **4 x 1 = 4 Marks**

   a) Define the terms (i) net work (ii) isomorphic graphs and give an example of each.

   b) Prove that every distributive lattice is modular

   c) Define the group Kernel of a monoid and find the group Kernels of the monoids $(IN,+)$ and $(Q,+)$

   d) Prove that the distance of a linear code is equal to the minimum weight of any nonzero codeword.