

# ANDHRA UNIVERSITY SCHOOL OF DISTANCE EDUCATION ASSIGNMENT QUESTION PAPER 2021-2022 M.A. / M.Sc. Mathematics (Final) COMPLEX ANALYSIS

## Note: Answer ALL Questions. All Questions carry equal marks.

Section - A

(4 x 4 =16 Marks)

- 1. (a) For a > 1 show that  $\int_0^{\pi} \frac{d\theta}{a+b \cos\theta} = \frac{\pi}{\sqrt{a^2-1}}$ .
  - (b) State and prove Residue theorem.
- 2. (a) State and prove Cauchy's integral formula (first version).
  - (b) Show that a Mobius transformation takes circles onto circles.
- 3. (a) State and prove Mittag Leffler's theorem.
  - (b) State and prove Schwarz reflection principle.
- 4. (a) State and prove Montel's theorem.
  - (b) State and prove Schwartz reflection principle.

Section - B

(4 x 1 = 4)

- (a) Define analytic function and Mobius transformation.
- (b) Define analytic function and Cauchy Riemann equations.
- (c) State and prove Schwarz's lemma.
- (d) State and prove Mean value theorem.



## ANDHRA UNIVERSITY SCHOOL OF DISTANCE EDUCATION ASSIGNMENT QUESTION PAPER 2021-2022 M.A. / M.Sc. Mathematics (Final) MEASURE THEORY AND FUNCTIONAL ANALYSIS

## Note: Answer ALL Questions. All Questions carry equal marks.

### Section - A

(4 x 4 =16 Marks)

- 1. (a) Show that the outer measure of an interval is its length.
  - (b) State and prove bounded convergence theorem.
- 2. (a) State and prove Radon Nikodym theorem.
  - (b) State and prove Riesz Representation theorem.
- 3. (a) State and prove Hahn Banach theorem.
  - (b) State and prove the closed graph theorem.
- 4. (a) State and prove Bessel's inequality.
  - (b) Let H be a Hilbert space, and let f be an arbitrary function in  $H^*$ . Then prove that there exists a unique vector y in H such that f(x) = (x, y) for every x in H.

### Section - B (4 x 1 = 4)

- (a) Prove that  $m^*$  is translation invariant.
- (b) Let  $\nu$  be a signed measure on the measurable space  $(X, \mathcal{B})$ . Suppose *E* is a measurable set such that  $0 < \nu(E) < \infty$ . Then prove that there is a positive set *A* contained in *E* with  $\nu(A) > 0$ .
- (c) Prove that in a normed linear space, addition and scalar multiplication are jointly continuous.
- (d) If x and y are any two vectors in a Hilbert space, then show that  $|(x, y)| \le ||x|| \cdot ||y||$ .



# ANDHRA UNIVERSITY SCHOOL OF DISTANCE EDUCATION ASSIGNMENT QUESTION PAPER 2021-2022 M.A. / M.Sc. Mathematics (Final) NUMBER THEORY

## Note: Answer ALL Questions. All Questions carry equal marks.

### Section - A

(4 x 4 =16 Marks)

- 1. (a) Let f be multiplicative. Then prove that f is completely multiplicative if and only if  $f^{-1}(n) = \mu(n) \cdot f(n)$  for all  $n \ge 1$ .
  - (b) State and prove Euler's Summation formula.
- 2. (a) For  $n \ge 1$  prove that  $\phi(n) = \prod_{p|n} \left(1 \frac{1}{n}\right)$ .
  - (b) If both g and  $f^*g$  are multiplicative, then prove that f is also multiplicative.
- 3. (a) Prove that there are infinitely many primes of the form 4n+1.
  - (b) State and prove Euler's Criterion.
- 4. (a) State and prove quadratic reciprocity law.
  - (b) State and prove Euler's criterion.

Section -B  $(4 \times 1 = 4)$ 

- (a) State and Prove Euler Fermet theorem.
- (b) If  $n \ge 1$  then prove that  $\log n = \sum_{d|n} \Lambda(d)$ .
- (c) State and prove Gauss Lemma.
- (d) Show that there are infinitely many primes of the form 4n-1.



# ANDHRA UNIVERSITY SCHOOL OF DISTANCE EDUCATION ASSIGNMENT QUESTION PAPER 2021-2022 M.A. / M.Sc. Mathematics (Final) LATTICE THEORY

### Note: Answer ALL Questions. All Questions carry equal marks.

#### Section - A

(4 x 4 =16 Marks)

- 1. (a) Define partial ordered set and order isomorphism. Prove that two finite partially ordered sets can be represented by the same diagram if and only if they are order isomorphic.
  - (b) Define a weakly complemented lattice and a semi complemented lattice. Prove that every uniquely complemented lattice is weakly complemented.
- 2. (a) Show that every element of a compactly generated lattice can be represented as a meet of a completely meet irreducible elements.
  - (b) Show that the dual, every sublattice and every homomorphic image of a modular lattice is modular.
- 3. (a) Show that in a Boolean algebra B, a proper ideal M is prime if and only if it is maximal.
  - (b) Show that in a section complemented lattice, every ideal constitutes the kernel of at most one congruence relation.
- 4. (a) Show that every complete Boolean algebra is infinitely distributive.
  - (b) Let *L* be a lattice. Then prove that *L* is distributive if and only if for any  $x \neq y \in L$ , there is a prime ideal containing one of then but not both.

#### Section - B (4 x 1 = 4)

- (a) Show that any interval of a lattice is a sub lattice.
- (b) Prove that any Boolean ring can be regarded as a Boolean algebra and Vice Versa.
- (c) Write about Boolean algebras and Boolean rings.
- (d) Write about congruence relation of lattices.



# ANDHRA UNIVERSITY SCHOOL OF DISTANCE EDUCATION ASSIGNMENT QUESTION PAPER 2021-2022 M.A. / M.Sc. Mathematics (Final) LINEAR PROGRAMMING AND GAME THEORY

## Note: Answer ALL Questions. All Questions carry equal marks.

Section - A

(4 x 4 =16 Marks)

- 1. (a) Show that for any matrix A, the row rank and column rank are equal.
  - (b) If *c* is a finite cone, then show that  $c^{**} = c$ .
- 2. (a) Define feasible solution, optimal solution of a standard maximization problem and state and prove optimality criterion for standard maximization problem.
  - (b) Solve the game with pay off matrix  $\begin{bmatrix} -5 & 5 & 0 & -1 & 8 \\ 8 & -4 & -1 & 6 & -5 \end{bmatrix}$ .
- 3. Find  $y = (n_1, n_2, ..., n_5) \ge 0$  which minimizes  $n_1 + 6n_2 7n_3 + n_4 + 5n_5$  subject to  $5n_1 4n_2 + 13n_3 2n_4 + n_5 = 20$ ,  $n_1 n_2 + 5n_3 n_4 + n_5 = 8$ .
- 4. (a) Show that a game  $\Gamma$  has at most one value.
  - (b) Show that the dual problems (A, b, c) have solutions if and only if  $\Gamma(A, b, c)$  has an optimal strategy  $I = (\partial_0, \partial_1, ..., \partial_{m+n})$  with  $\partial > 0$ . In this case show that there is a One to one correspondence between such strategies and solutions of (A, b, c).

Section - B 
$$(4 x 1 = 4)$$

- (a) Write about the optimality criterion for simplex method.
- (b) Explain the concept of maximum flow problem.
- (c) Define (i) Standard maximum problem (ii) Canonical maximum problem
- (d) Define (i) convex set (ii) convex hull (iii) convex polytope.



# ANDHRA UNIVERSITY SCHOOL OF DISTANCE EDUCATION ASSIGNMENT QUESTION PAPER 2021-2022 M.A. / M.Sc. Mathematics (Final) UNIVERSAL ALGEBRA

## Note: Answer ALL Questions. All Questions carry equal marks.

### Section - A

(4 x 4 =16 Marks)

- 1. (a) Prove that L is non distributive lattice iff  $M_5$  or  $N_5$  can be embedded into L.
  - (b) If A is congruence permutable, then show that A is congruence modular.
- 2. (a) Let *P* be a poset such that  $\land A$  exists for every subset *A* of *P* or such that  $\lor A$  exists for every subset of *A* of *P*. Then prove that *P* is a complete lattice.
  - (b) Prove that every algebraic lattice is isomorphic to the lattice of closed subsets of some set A with algebraic closure operator C.
- 3. (a) If A is congruence permutable, then show that A is congruence modular.
  - (b) Let  $\alpha: A \to B$  be a homomorphism. Then prove that  $Ker(\alpha)$  is a congruence on A.
- 4. (a) Show that if L is a sub directly irreducible distributive lattice then  $|L| \le 2$ .
  - (b) State and prove Stone Duality theorem.

### Section – B $(4 \times 1 = 4)$

- (a) Define a modular lattice. Show that every distributive lattice is a modular lattice.
- (b) Define the terms (i) Lattice (ii) Partially ordered set.
- (c) State and prove Second isomorphism theorem.
- (d) Let X be a set. Then show that  $S \cup (X) \cong 2^X$ .



# ANDHRA UNIVERSITY SCHOOL OF DISTANCE EDUCATION ASSIGNMENT QUESTION PAPER 2021-2022 M.A. / M.Sc. Mathematics (Final) INTEGRAL EQUATIONS

### Note: Answer ALL Questions. All Questions carry equal marks.

Section - A

( 4 x 4 =16 Marks)

1. (a) Solve the following symmetric integral equation with the help of Hilbert - Schmidt theorem.

 $y(x) = 1 + \lambda \int_0^{\pi} \cos(x+t) y(t) dt.$ 

- (b) Solve  $y'(t) = t + \int_0^1 y(t-x) \cos x \, dx$ , y(0) = 4.
- 2. (a) State and prove Hilbert Schmidt Theorem.
  - (b) Transform the initial value problem  $\frac{d^2y}{dx^2} + y = \cos x$ , y(0) = 0, y'(0) = 1 into an integral equation.
- 3. (a) Using Green's function, Solve the boundary value problem y'' y = x, y(0) = y(1) = 0.
  - (b) Solve  $y(x) = \cos x x 2 + \int_0^x (t x)y(t)dt$ .
- 4. (a) Find the eigen values and eigen functions of the homogeneous integral equation  $y(x) = \lambda \int_{1}^{2} \left( xt + \frac{1}{xt} \right) y(t) dt.$ 
  - (b) Find first and second approximations in the iterative solution of the integral equation  $\int_0^1 (x+y)^{\frac{1}{2}} [\varphi(y)]^{\frac{1}{2}} dy = \varphi(x).$

### Section - B (4 x 1 = 4)

- (a) Define Fredholm integral equation of the first and second kind.
- (b) Write the four properties to construct the Green's functions.
- (c) Describe the shop stocking problem.

(d) Solve 
$$y(x) = 1 + \int_0^x y(t) dt$$
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# ANDHRA UNIVERSITY SCHOOL OF DISTANCE EDUCATION ASSIGNMENT QUESTION PAPER 2021-2022 M.A. / M.Sc. Mathematics (Final) COMMUTATIVE ALGEBRA

## Note: Answer ALL Questions. All Questions carry equal marks.

### Section - A

( 4 x 4 =16 Marks)

- 1. (a) Prove that the set R of all nilpotent elements in a ring A is an ideal, and A/R has no nilpotent element  $\neq 0$ .
  - (b) State and prove Nakayama's lemma.
- 2. (a) State and prove second uniqueness theorem.
  - (b) State and prove first uniqueness theorem.
- 3. (a) Prove that the length l(M) is an additive function on the class of A- modules of finite length.
  - (b) State and prove going down theorem.
- 4. (a) State and prove Hilbert basis theorem.
  - (b) Prove that in a Noetherian ring every irreducible ideal is primary.

### Section - B (4 x 1 = 4)

- (a) Show that  $S^{-1}(A)$  is a flat A-module.
- (b) Show that M has a composition series if and only if M satisfies both chain conditions.
- (c) Prove that in a Noetherian ring A, every ideal a contains a power of its radical.
- (d) If I = r(I), then prove that I has no embedded prime ideals.



## ANDHRA UNIVERSITY SCHOOL OF DISTANCE EDUCATION ASSIGNMENT QUESTION PAPER 2021-2022 M.A. / M.Sc. Mathematics (Final) NUMERICAL ANALYSIS AND COMPUTER TECHNIQUES

## Note: Answer ALL Questions. All Questions carry equal marks.

1

#### Section - A

(4 x 4 =16 Marks)

- (a) Write the difference between FUNCTION and SUBROUTINE.
  - (b) Give the computed value of the following FORTRAN EXPRESSION. Using I = -2, J = 5, K = -2, A = 10.5, B = 2.5.
    i). J \*\* I / J \* K ii). A \*\* I - B.
- 2. (a) Write a Fortran program to evaluate the integral  $\int_{1}^{2} \sin 2x \, dx$  using simpson's rule with 4 sub intervals.
  - (b) Write a FORTRAN program to find the transpose of a 3X3 matrix.
- 3. (a) Solve the boundary value problem u'' = u + x, u(0) = 0, u(1) = 0 with  $h = \frac{1}{4}$  use the numerov method.
  - (b) Evaluate the integral  $I = \int_{-1}^{1} (1 x^2)^{\frac{3}{2}} \cos x \, dx$  by using Gauss Legendre 3 point formula.
- 4. (a) Solve the initial value problem  $u' = -2tu^2$ , u(0) = 1 with h = 0.2 on the interval [0,1]. Use the fourth order classical Runge Kutta Method.
  - (b) Using the following data, find f'(6.0).

х	6.0	6.1	6.2	6.3	6.4
f(x)	0.1750	-0.1998	-0.2223	-0.2422	-0.2596

Section - B

(4 x 1 = 4)

- (a) Write about the Variable with examples.
- (b) Explain shooting method.
- (c) Write about single step method.
- (d) Explain the third order Runge Kutta method.