ANDHRA UNIVERSITY
SCHOOL OF DISTANCE EDUCATION
ASSIGNMENT QUESTION PAPER 2019-2020
M.A. / M.Sc (Previous) Mathematics
ALGEBRA

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Prove that a homomorphism $\phi : G \to H$ is injective if and only if $ker\ \phi = \{e\}$.

   (b) State and prove Cayley's theorem

2. (a) Prove that alternating group A_n is simple if $n > 4$. Consequently S_n is not solvable if $n > 4$.

   (b) State and prove Cauchy's theorem for abelian group.

3. (a) Let $f : R \to S$ be a homomorphism of a ring R into a ring S. Then prove that $ker\ f = (0)$ if and only if f is 1-1.

   (b) If R is a commutative ring, then prove that an ideal P in R is prime if and only if $ab \in P,\ a \in R,\ b \in R,$ implies $a \in P\ or\ b \in P$.

4. (a) Let $f(x) \in \mathbb{Z}[x]$ be prime. Then prove that $f(x)$ is reducible over Q if and only if $f(x)$ is reducible over $\mathbb{Z}$.

   (b) Let E and F be fields and let $\sigma : F \to E$ be an embedding of F into E. Then prove that $\exists$ a field K such that F is a subfield of K and σ can be extended to an isomorphism of K onto E.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

(a) Let G be a group and $a, b \in G$ such that $ab = ba$. If $o(a) = m,\ o(b) = n$ and $(m,n) = 1$ then prove that $o(a,b) = mn$.

(b) Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 5 & 1 & 6 & 4 & 3 \end{pmatrix}$ as a product of disjoint cycles.

(c) Prove that the centre of a ring is a subring.

(d) Find the smallest extension of Q having a root of $x^2 + 4 \in Q[x]$.

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ASSIGNMENT QUESTION PAPER 2019-2020
M.A. / M.Sc (Previous) Mathematics
LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Let T be a linear operator on an $n-$ dimensional vector space V. Then prove that the characteristic and minimal polynomials for T have the same roots, except for multiplicities.

   (b) State and prove Cayley - Hamilton theorem.

2. (a) If $y_1(x)$ and $y_2(x)$ are any two solutions of $y''+p(x)y'+Q(x)y=0$ on $[a,b]$, then prove that their Wronskian $W=W(y_1,y_2)$ is either identically zero or never zero on $[a,b]$.

   (b) Use method of variation of parameters, solve $y''+y=cosec$.

3. (a) If $W(t)$ is the Wronskian of the two solutions $x=x_1(t),\ y=y_1(t)$ and $x=x_2(t),\ y=y_2(t)$ of the homogeneous system $\dfrac{dx}{dt}=a_1(t)\,x+b_1(t)\,y;$ $\dfrac{dy}{dt}=a_2(t)\,x+b_2(t)\,y\rightarrow(1)$ are linearly independent on $[a,\,b]$, then prove that $x=c_1x_1(t)+c_2x_2(t);\ \mathrm{y}=\mathrm{c}_1\mathrm{y}_1(t)+c_2y_2(t)$ is the general solution of system (1) on this interval.

   (b) Find the general solution of the system $\dfrac{dx}{dt}=-3x+4y;\ \dfrac{dy}{dt}=-2x+3y$.

4. (a) By the method Laplace transforms, find the solution of

   $y''-4y'+4y=0,\ y(0)=0\ and\ y'(0)=3.$

   (b) State and prove convolution theorem on Laplace transforms.

Section - B **(4 x 1 = 4)**

5. Answer all the following

(a) Let $T \in L(R), F = \mathbb{R}$ and matrix of T w.r.t. the standard basis is $\begin{bmatrix} 5 & 3 \\ -6 & -4 \end{bmatrix}$. Find the characteristic and minimal polynomials of T.

(b) Consider two functions $f(x) = x^3$ and $g(x) = x^2|x|$ on the interval $[-1, 1]$. Show that their Wronksian $W(f, g)$ vanishes identically.

(c) Find the general solution of the system

$$\begin{cases} \dfrac{dx}{dt} = x \\ \dfrac{dy}{dt} = y \end{cases}$$

(d) Find the inverse Laplace transforms of $\dfrac{12}{(p+3)^4}$.

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ASSIGNMENT QUESTION PAPER 2019-20
M.A. / M.Sc. (PREVIOUS) MATHEMATICS
REAL ANALYSIS
Answer ALL Questions
All Questions carry equal marks

SECTION – A **(4 X 4 = 16 Marks)**

1(a) If $\bar{E}$ is the closure of a set E in a metric space X, then prove that diam $\bar{E}$ = diam E.

(b) Suppose f is a continuous real function on a compact metric space X, and $M = sup_{p\in X}\ f(p)$, $m = \inf_{p\in X} f(p)$. Then prove that there exist points $p,\ q \in X$ such that $f(p) = M$ and $f(q) = m$.

2 (a) Assume α increases monotonically and $\alpha' \in R$ on [a, b]. Let f be a bounded real function on [a,b]. Then prove that $f \in R(\alpha)$ if and only if $f\alpha' \in R$. In that case $\int_a^b f\, d\alpha = \int_a^b f(x)\alpha'(x)dx$.

(b) Suppose ϕ is strictly increasing continuous function that maps an interval [A, B] onto [a, b]. Suppose α is monotonically increasing on [a, b]and $f \in R(\alpha)$ on [a, b]. Define β and g on [A, B] by$\beta(y) = \alpha(\phi(y))$, $g(y) = f(\phi(y))$. Then prove that $g \in R(\beta)$ and $\int_A^B g\, d\beta = \int_a^b f\, d\alpha$.

3 (a) Suppose$\{f_n\}$ is a sequence of functions, differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$, then prove that $\{f_n\}$ converges uniformly on $[a, b]$, to a function f and $f'(x) = \lim_{n\to\infty} f_n'(x)$, $(a \le x \le b)$.

(b) Suppose $\sum c_n$ converges. Put $f(x) = \sum_{n=0}^{\infty} c_n x_n \ (-1 < x < 1)$. Then prove that

$$\lim_{x\to 1} f(x) = \sum_{n=0}^{\infty} c_n .$$

4 (a) IfX is a complete metric space, and if φ is a contraction of x into X, then prove that there exists one and only one $x \in X$ such that $\varphi(x) = x$.

(b) State and prove implicit function theorem.

SECTION – B **(4 X 1 = 4 Marks)**

Answer all the following:

5 (a) If $0 \le x < 1$, then prove that $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$.

(b) If $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a, b]$ then prove that $f + g \in R(\alpha)$ on $[a, b]$ and

$$\int_a^b (f + g)d\alpha = \int_a^b f\, d\alpha + \int_a^b g\, d\alpha .$$

(c) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

(d) If $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & for\ \ (x,y) \ne (0,0) \\ 0 & for\ (x,y) = (0,0) \end{cases}$ then prove that $(D_1 f)(x, y)$ and $(D_2 f)(x, y)$ exist at every point of $\mathbb{R}^2$, although f is not continuous at (0,0)

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ASSIGNMENT QUESTION PAPER 2019-2020
M.A. / M.Sc (Previous) Mathematics
TOPOLOGY

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Let X be a metric space then prove that a subset G of X is open $\Leftrightarrow$ it is a union of open spheres.

   (b) Let X and Y be metric spaces and f a mapping of X into Y. Then prove that f is continuous $\Leftrightarrow$ $f^{-1}(G)$ is open in X whenever G is open in Y.

2. (a) State and prove Lindelof 's theorem.

   (b) Prove that every sequentially compact metric space in totally bounded.

3. (a) Prove that every compact Hausdorff space is normal.

   (b) Let X be a Hausdorff space. If X has an open base whose sets are also called, then prove that X is totally disconnected.

4. (a) State and prove real Stone - Weirstrass theorem.

   (b) Show that a Hausdorff space is locally compact if and only if each of its points in an interior point of some compact space.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

(a) Let X be an arbitrary non-empty set, and define d by $d(x,y)=\begin{cases}0 \; if \; x=y \\ 1 \; if \; x\neq y\end{cases}$. Then prove d is a metric on X.

(b) Show that a subspace of a topological space is itself a topological space.

(c) Show that any continuous image of a compact space is compact.

(d) If X is a locally compact Hausdorff space, then prove that $C_o(X,R)$ is a sublattice of $C(X,R)$.

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ASSIGNMENT QUESTION PAPER 2019-2020
M.A. / M.Sc (Previous) Mathematics
DISCRETE MATHEMATICS
Answer ALL Questions
All questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Prove that a graph is bipartite if and only if it contains no odd cycles.

   (b) Let $G(V,E)$ be a graph with no isolated vertex. Then show that G has an Euler circuit if and only if G is connected and every vertex of G is even.

2. (a) Show that every distributive lattice is modular. Is the converse of this result true? Justify your claim.

   (b) Let B be a Boolean algebra. Prove that an ideal M in B is maximal if and only if for any $b \in B$ either $b \in M$ or $b' \in M$, but not both hold.

3. (a) Describe an automaton and semi automaton. Describe the cafeteria automaton and draw the state graph of this automaton.

   (b) Explain by means of an example the concept of an automaton associated with a monoid $(S, \bullet)$. Show that there exists an automaton whose monoid is isomorphic to $(S, \bullet)$.

4. (a) State and prove the Hamming bound theorem.

   (b) Let C be an ideal $\neq \{0\}$ *of* V_n. Then prove that there exists a unique $g \in V_n$ with the following properties.

   (i) $g \mid x^n - 1 \ in \ F_q[x]$ (ii) $C = (g)$ (iii) g is monic.

Section - B **(4 x 1 = 4)**

5. Answer all the following :

(a) Show that a graph is a tree if and only if it has no cycles and $|E| = |V| - 1$.

(b) Determine the symbolic representation of the circuit given by

$$p = (x_1 + x_2 + x_3)(x' + x_2)(x_1x_3 + x_1' x_2)(x_2' + x_3)$$

(c) Define the group kernel of a monoid (S, o). Show that the group kernel G_5 is a group within (S, o).

(d) Show that a linear code $C \subseteq V_n$ is cyclic if and only if C is an ideal in V_n.

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M.A. / M.Sc (Final) Mathematics
COMPLEX ANALYSIS

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Let Z_1, Z_2, Z_3, Z_4 be four distinct points in $\mathbb{C}_\infty$. Then prove that (Z_1, Z_2, Z_3, Z_4) is a real number iff all four points lie on a circle.

   (b) Define Mobius transformation and show that every Mobius transformation is the composition of translation, dilations and the inversion.

2. (a) For $a > 1$, show that $\int_0^\pi \frac{d\theta}{a + cos\theta} = \frac{\pi}{\sqrt{a^2 - 1}}$

   (b) State and prove Residue theorem.

3. (a) State and prove Montel's theorem

   (b) State and prove Arzela - Ascoli theorem.

4. (a) State and prove Mittag - Leffler's theorem.

   (b) State and prove Schwarz reflection principle

Section - B **(4 x 1 = 4)**

5. Answer all the Following :

   (a) Define analytic function and Mobius transformation.

   (b) Let $u(x, y) = e^x cos\ y$. Show that there exists an analytic function f such that $Ref = u$.

   (c) State and prove Schwarz'a Lemma.

   (d) State and prove Mean-value theorem

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ASSIGNMENT QUESTION PAPER 2019-2020
M.A. / M.Sc (Final) Mathematics
Measure Theory and Functional Analysis
Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Show that the outer measure of an interval is its length.

   (b) State and prove Bounded Convergence theorem.

2. (a) State and prove Fatou's lemma.

   (b) State and prove Radon-Nikodym theorem.

3. (a) State and prove Hahn – Banach theorem.

   (b) State and prove the closed graph theorem.

4 (a) State and prove Gram-Schmidthorthogonalisation process.

   (b) Show that an operator T on H is unitary if and only if it is an isometric isomorphism of H onto itself.

SECTION – B

(4 X 1 = 4 Marks)

Answer all the following :

5 (a) Prove that m^* is translation invariant.

   (b) Let (X, B, μ) be a finite measure space and g an integrable function such that for some constant M, $\left|\int g\varphi \, db\right| \leq M \, \|\varphi\| p$ for all simple functions φ. Then show that $g \in Lq$.

   (c) State and prove Holder's inequality.

   (d) If P is the projection on a closed linear subspace M of , then show that reduces an operator if and only if $T \Leftrightarrow TP = PT$.

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M.A. / M.Sc (Final) Mathematics
NUMBER THEORY

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Let f be multiplicative. Then f is completely multiplicative if and only if

   $f^{-1}(n)=\mu(n)f(n)$, for all $n \geq 1$.

   (b) State and prove Euler's summation formula

2. (a) Let P_n denote the n[th] prime. Then show that the following relations are equivalent

   (i) $\lim_{x\to\infty} \frac{\pi(x)\, log\ x}{x} = 1$

   (ii) $\lim_{x\to\infty} \frac{\pi(x)\, log\ (\pi(x))}{x} = 1$.

   (iii) $\lim_{n\to\infty} \frac{\rho_n}{n\ log^n} = 1$

   (b) Prove that, for every integer $n \geq 2$ $\frac{1}{6}\frac{n}{log\, n} < \pi(n) < 6\ \frac{n}{log\ n}$

3. (a) Prove that there are infinitely many primes of the form $4n+1$.

   (b) For $n > 1$ and $\chi \neq \chi_1$, prove that

   $$\sum_{P \leq x} \frac{\chi(p) log\ p}{p} = -L^{-1}\ (1, \chi). \sum_{n \leq x} \frac{\mu(n)\chi(n)}{n} + 0(1)$$

4. (a) State and prove Euler's Criterion.

   (b) In ψ is any Dirichlet character mod k, then prove that $G(n, \chi) = \overline{\chi(n)}\ G(1, \chi)$, whenever $(n, k) = 1$.

Section - B **(4x1=4)**

5. Answer all the Following :

(a) For $x > 0,$ show that

$$0 \le \frac{\psi(x)}{x} - \frac{\theta(x)}{x} \sum \frac{(log\, x)^2}{2\sqrt{x}\ log\ 2}$$

(b) State and prove Euler - Fermet theorem

(c) Let χ be any real valued character m®od k and let $A(n) = \sum_{d/n} \chi(d)$. Then prove that $A(n) \ge 0$ for all n and $A(n) \ge 1,$ if n is a square.

(d) If p and q are odd positive integers then show that $(m/p)(n/p) = (mn/p)$.

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M.A. / M.Sc (Final) Mathematics
(Optional P - II) LATTICE THEORY
Answer ALL Questions
All Questions carry equal marks

Section - A **(4 x 1 =4 Marks)**

1. (a) Define partially ordered set and order isomorphism. Prove that two finite partially ordered sets can be represented by the same diagam if and only if they are order isomorphic.

   (b) Define a weakly complemented lattice and a semi complemented lattice. Prove that every uniquely complemented lattice is weakly complemented.

2. (a) State and prove the lattice theoritical duality principle.

   (b) Let L be a distributive lattice. Then show that the lattice $\left(I(L), \leq\right)$ of all ideals of L is a distributive lattice.

3. (a) For a complete Boolean algebra B show that the following are equivalent.

   (i) B is completely meet-distributive

   (ii) B is completely join distributive

   (iii) B is atomic

   (iv) B is dually atomic

   (b) Let $(B, +, \bullet)$ be a Boolean algebra then $R(B(R)) = R.$

4. (a) Show that in a Boolean algebra $B,$ a proper ideal M of B is prime if and only if it is maximal.

   (b) Show that in a section complemented lattice, every ideal constitutes the kernel of atmost one congruence relation.

Section - B

5. Answer all the Following :

(a) Show that any interval of a lattice is a sub lattice.

(b) Show that a modular lattice satisfies both the upper and lower covering conditions.

(c) Prove that any Boolean ring can be regarded as a Boolean algebra and Vice - Versa.

(d) If θ is a congruence relation on L then prove that every θ class is a convex sublattice of $L.$

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ASSIGNMENT QUESTION PAPER 2019-2020
M.A. / M.Sc (Final) Mathematics
Optional - LINEAR PROGRAMMING AND GAME THEORY
Answer ALL Questions
All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Show that for any matrix A, the row rank and column rank are equal.

   (b) If c is a finite cone, then show that $c^{**} = c.$

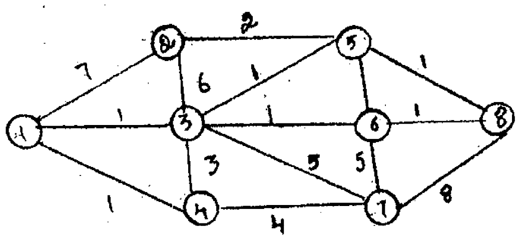
2. (a) If both a program and its dual are feasible then show that both have optimal vectors and the values of the two programs are the same. If either program is infeasible then neither has an optimal vector.

   (b) Find $y = (n_1, n_2, ..., n_5) \geq 0$ which minimizes $n_1 + 6n_2 - 7n_3 + n_4 + 5n_5$ subject to

   $5n_1 - 4n_2 + 13n_3 - 2n_4 + n_5 = 20$, $n_1 - n_2 + 5n_3 - n_4 + n_5 = 8$

3. (a) In the network below, nodes (1) and (6) are sources with supplies $\sigma(1) = 3, \sigma(6) = 5.$

   Nodes (4) and (8) are sinks with demands $\delta(4) = 4, \delta(8) = 4.$ The numbers on the edges are capacities which are assumed to be the same in both directions. Determine whether this transhipment problem is feasible.



   (b) Find a soultion of the optimal assignment problem whose rating matrix is the following.

0	15	9	1	3	4	19
2	0	19	11	9	3	14
12	5	17	12	24	15	16
19	11	14	23	16	17	29
20	15	23	22	19	21	24
23	12	16	17	24	25	26
25	16	8	26	21	20	23

4. (a) Show that a game Γ has at most one value.

(b) Show that the dual problems $(A, \mathrm{b}, \mathrm{c})$ have solutions if and only if $\Gamma(A, \mathrm{b}, \mathrm{c})$ has an optimal strategy $I = (\partial_0, \partial_1, ... \partial_{\mathrm{m}+n})$ with $\partial_n > 0$. In this case show that there is a One-to-one correspondence between such strategies and solutions of $(A, \mathrm{b}, \mathrm{c})$.

Section - B (4 x 1 = 4)

5. Answer all the Following :

(a) Define

(i) convex set

(ii) convex hull

(iii) convex polytope.

(b) Write about the optimality criterion for simplex method.

(c) Explain the concept of maximum - flow problem.

(d) Define the terms

(i) Two person zero - sum game

(ii) Matrix game

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M.A. / M.Sc (Final) Mathematics

UNIVERSAL ALGEBRA

Answer ALL Questions

All Questions carry equal marks

Section - A **4 x 4 = 16**

1. (a) Prove that L is non distributive lattice iff M_5 or N_5 can be embedded into L.

   (b) If for $\theta \in E_9(A)$ and $a, b \in A$ then show that

   (i) $A = \bigcup_{\alpha \in A} a/\theta$ (ii) $a/\theta \neq b/\theta$ implies $a/\theta \cap b/\theta = \phi$.

2. (a) If A is congruence permutable, then show that A is congruence modular.

   (b) Let $\alpha : A \to B$ be a homomorphism. Then prove that $Ker(\alpha)$ is a congruence on A.

3. (a) If for an indexed family of maps $\alpha_i : A \to A_i, i \in I,$ then show that the following are equivalent.

   (i) The maps x_i seperate points

   (ii) α is injective

   (iii) $\bigcap_{i \in I} Ker\, \alpha_i = \Delta$

   (b) Suppose $U_1(X_1)$ and $U_2(X_2)$ are two algebras in a class K with the universal mapping property for K with the universal mapping property for K over the indicated sets. $|X_1| = |X_2|$, then show that $U_1(X_1) \cong U_2(X_2)$.

4. (a) Let $B = \langle B, V, \wedge, \bullet, 0, 1 \rangle$ be a Boolean algebra. Define $B^{\otimes}$ to be the algebra $\langle B, +, \bullet, -, 0, 1 \rangle$, where $a + b = (a \wedge b') \vee (a' \wedge b)$ $a \wedge b = a \bullet b$, a' = 1 + a. Then show that $B^{\otimes}$ is a Boolean ring.

   (b) Let $V(K)$ be a congruence distributive variety. If A is a subdirectly irreducible algebra in $V(K)$, then prove that $A \in HSP_U(K)$; hence $V(K) = IP_S HSP_U(K)$.

Section - B **(4 x 1 =4 Marks)**

5. Answer all the Following :

(a) Define the terms

(i) Lattice (ii) Partially ordered set.

(b) If $\alpha : A \rightarrow B$ is an embedding, then prove that $\alpha\left(A\right)$ is a subuniverse of B.

(c) If B is a Boolean algebra and $a, b \in B$ then show that $N_a \cup N_b = N_{a \vee b}$, $N_a \cap N_b = N_{a \wedge b}$ and $N_{a'} = \left(N_a\right)'$. Thus in particular the $N_a's$ form a basis for the topology of B^*.

(d) Let X be a set. Then show that $SU\left(X\right) \cong 2^X$.

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M.A. / M.Sc (Final) Mathematics

Optional - INTEGRAL EQUATIONS

Answer ALL Questions

All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Form an integral equation corresponding to the differential equation

$\left(d^2y/dx^2\right) - sin\ x\left(dy/dx\right) + e^x y = x,$ with the inital conditions

$y(0) = 1,\ y'(0) = -1$.

(b) Solve $y'(t) = t + \int_0^1 y(t-x)\ cos\ xd\,x, \mathrm{y}(0) = 4$

2. (a) Solve the following symmetric integral equation with the help of Hilbert - Schmidt theorem.

$$y(x) = 1 + \lambda \int_0^{\pi} cos\ (x+t)\,y(t)\,dt$$

(b) Using Green's function, solve the boundary value problem $y'' - y = x,\ y(0) = y(1) = 0.$

3. (a) Find the resolvent Kernel of the Volterra integral equation with the Kernel $K(x,t) = (2 + cos\ x)/\ (2 + cos\ t)$.

(b) With the help of finite Hilbert transform, solve $x2 = \int_{-l}^{l} \frac{2ty(t)}{x^2 - t^2}\,dx,$ assuming that $Y(t) = -Y(-t)$.

4. (a) Solve $y(x) = cos\,x - x - 2 + \int_0^x (t-x)\ y(t)\ dt$

(b) Find the eigen values and eigen functions of the homogeneous integral equation $y(x) = \lambda \int_1^2 \left(xt + \frac{1}{xt}\right) y(t)\ dt$

Section - B **(4 x 1 = 4)**

5. Answer all the Following :

(a) Define Fredholm integral equations of the first and second kind.

(b) Write the four properties to construct the Green's functions.

(c) Solve the $y(x) = 1 + \int_0^x y(t)\,dt$

(d) Using the method of successive approximation, solve the integral equation $y(x) = 1 + x - \int_0^x y(t)\,dt,$ taking $y_0(x) = 1.$

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ASSIGNMENT QUESTION PAPER 2019-2020
M.A. / M.Sc (Final) Mathematics
COMMUTATIVE ALGEBRA
Answer ALL Questions
All Questions carry equal marks

Section - A **(4 x 1 =4 Marks)**

1. (a) Prove that the set R of all nilpotent elements in a ring A is an ideal, and A/R has no nilpotent element $\neq 0$.

   (b) State and prove Nakayama's lemma.

2. (a) If N, P are submodules of an $A-$ module M then

   (i) $S^{-1}(N+P) = S^{-1}(N) + S^{-1}(P)$

   (ii) $S^{-1}(N \cap P) = S^{-1}(N) \cap S^{-1}(P)$

   (iii) the $S^{-1}A-$ modules. $S^{-1}(M/N)$ and $(S^{-1}M)/(S^{-1}N)$ are isomorphic.

   (b) State and prove first uniqueness theorem.

3. (a) Let S be a multiplicatively closed subset of A, and let q be a primary ideal.

   (i) If $S \cap P \neq \phi$, then $S^{-1}q - S^{-1}A$.

   (ii) If $S \cap P = \phi$ then $S^{-1}q$ is $S^{-1}P-$ primary and its contraction in A is q.

   (b) State and prove going down theorem

4. (a) State and prove Hilbert basis theorem.

   (b) Prove that, In an Artin ring the nil radical R is nilpotent.

Section - B **(4x1=4)**

5. Answer all the Following :

(a) Show that $x \in R \Leftrightarrow 1 - xy$ is a unit in A for all $y \in A$.

(b) Prove that every ideal in $S^{-1}A$ is an extended ideal, where S is a multiplicative subset of A.

(c) Let q be a primary ideal in a ring A. Then show that $r(q)$ is the smallest prime ideal containing q.

(d) Show that M has a composition series $\Leftrightarrow M$ satisfies both chain conditions.

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ASSIGNMENT QUESTION PAPER 2019-2020
M.A. / M.Sc (Final) Mathematics
NUMERICAL ANALYSIS AND COMPUTER TECHNIQUES
Answer ALL Questions
All Questions carry equal marks

Section - A **(4 x 4 =16 Marks)**

1. (a) Write a program to evaluate double integral $\int_0^1 \left(\int_0^2 \frac{2xy}{(1+x^2)(1+y^2)^{dy}} \right) dx$ using the Simpson's rule with $h = k = 0.25$.

   (b) Write a program to find out the simple root of $f(x) = 0$ using Bisection method.

2. (a) Find the approximate value of the integral $I = \int_0^1 \frac{dx}{1+x}$ using Composite trapezoidal rule with 2, 3, 5, 9 nodes and Romberg integration.

   (b) For the method $f'(x_0) = -3\, \frac{f(x_0) + 4f(x_1) - f(x_2)}{2h} + \frac{h^2}{3} f^{m}(\xi) \quad x_0 < \xi < x_2$ determine the optimal value of h, using the criteria

   (a) $|RE| = |TE|$ (b) $|RE| + |TE| =$ minimum.

3. (a) Solve the initial value problem $u' = -2tu^2, \mathrm{u}(0) = 1$ with $h = 0.2$ on the interval $[0,1]$. Use the fourth order classical Runge - Kutta method.

   (b) Solve the boundary value problem $u'' = u + x, u(0) = 0, u(1) = 0$ with $h = \frac{1}{4}$, Use the numerov method.

4. (a) Solve the write a program to solve the initial value problem $u' = -2tu^2,\ u(0) = 1$ with $h = 0.2$ on the interval $[0,\ 0.4]$ using the predictor - corrector method.

   (b) Write the program to find the numerical solution at $x = 0.8$ for $\frac{dy}{dx} = \sqrt{x+y},\ y(0.4) = 0.41$ with $h = 0.2$ by Runge - Kutta formula of fourth order.

Section - B **(4 x 1 = 4)**

5. Answer all the Following :

(a) Find the approximate value of $I = \int_0^1 \frac{dx}{1+x}$ using trapezoidal rule.

(b) Explain the third order Runge - Kutta method.

(c) Solve the initial value problem $u' = -2tu^2, u(0) = 1$ with $h = 0.2$ on the interval $[0,1]$ using the backward Euler method.

(d) Write about single step method.