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M 309 --- NUMBER THEORY-I

18/19  
2005-06 AB

UNIT—I: ARITHMETICAL FUNCTIONS AND DIRICHLET MULTIPLICATION

Introduction , The Mobius function  $\mu(n)$ , The Euler totient function  $\varphi(n)$ , A relation connecting  $\varphi$  and  $\mu$ , A product formula for  $\varphi(n)$ , The Dirichlet product of arithmetical functions, Dirichlet inverses and Mobius Inversions formula, The Mangoldt function  $\Lambda(n)$ , Multiplicative functions, Multiplicative functions and Dirichlet multiplication, The inverse of a completely multiplicative function, Liouville's function  $\lambda(n)$ , The divisor functions  $\sigma$   $\alpha(n)$ , Generalised convolutions.

UNIT—II AVERAGES OF ARITHMETICAL FUNCTIONS

Introduction, The big oh notation Asymptotic equality of functions, Euler's summation formula, some elementary asymptotic formulas, the average order of  $d(n)$ , the average order of divisor functions  $\sigma_{\alpha}(n)$ , the average order of  $\varphi(n)$ , An application to the distribution of lattice points visible from the origin, The average order of  $\mu(n)$  and  $\Lambda(n)$ , The partial sums of a Dirichlet product, Applications to  $\mu(n)$  and  $\Lambda(n)$ , Another identity for the partial sums of a Dirichlet product.

UNIT—III SOME ELEMENTARY THEOREMS ON THE DISTRIBUTION OF PRIME NUMBERS

Introduction , Chebyshev's functions  $\psi(x)$  and  $\vartheta(x)$ , Relations connecting  $\psi(x)$  and  $\vartheta(x)$ , some equivalent forms of the prime number theorem, Inequalities of  $\pi(n)$  and  $P_n$ , Shapiro's Tauberian theorem, Application of Shapiro's theorem, An asymptotic formulae for the partial sums  $\sum_{p \leq x} (1/p)$ ,

UNIT—IV CONGRUENCES

Definition and basic properties of congruences, Residue classes and complete residue systems, Linear congruences, Reduced residue systems and Euler- Fermat theorem, Polynomial congruences modulo  $p$ , Lagrange's theorem, Simultaneous linear congruences, the Chinese remainder theorem, Applications of the Chinese remainder theorem, Polynomial congruences with prime power moduli.

CONTENT AND EXTENT AS IN THE TEXT BOOK.

INTRODUCTION TO ANALYTIC NUMBER THEORY - BY TOM M. APOSTOL,  
NAROSA PUBLISHING HOUSE, NEW DELHI

(Sections 2.2 to 2.14, 3.1 to 3.12, 4.1 to 4.9, 5.1 to 5.9.)