M.Sc Degree Examination
First Semester
Applied Mathematics
21AM101: Real Analysis
(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1. (a) Let S$\subset \mathbb{R}^n$. Then show that following are equivalent
   (i) S is Compact.
   (ii) S is closed and bounded.
   (iii) Every infinite subset of S has a limit point in S.

   (b) Let P be a non empty perfect set in R^k. Then show that P is uncountable.

OR

2. (a) Let $\{E_n\}, n = 1,2,3,...$ be a sequence of countable sets, then prove that $\bigcup_{n=1}^{\infty} E_n$ is a countable set.

   (b) Prove that the function defined below is discontinuous everywhere.

$$f(x) = \begin{cases} 1, & x \text{ rational,} \\ 0, & x \text{ irrational.} \end{cases}$$

UNIT-II

3. (a) Let α is increasing on $[a, b]$. If $f \in R(\alpha)$ on $[a, b]$, then show that$f^2 \in R(\alpha)$ on $[a, b]$.

   (b) If $f \in R(\alpha)$ on $[a, b]$, then show that $\alpha \in R(f)$ and also show that $\int_a^b f(x)d\alpha(x) + \int_a^b \alpha(x)df(x) = f(b)\alpha(b) - f(a)\alpha(a)$.

OR

4. (a) If $f \in R(\alpha), g \in R(\alpha)$ on $[a, b]$, then show that $C_1 f + C_2 g \in R(\alpha)$ where C_1 and C_2 are constants on $[a, b]$.

   (b) Let α is increasing on $[a, b]$. Then for any two partitions P_1 and P_2 , prove that $L(P_1, f, \alpha) \leq U(P_2, f, \alpha)$.

UNIT-III

5. (a) If $f \in R$ and $g \in R$ on $[a, b]$, let $F(x) = \int_a^x f(t)dt$, and $G(x) = \int_a^x g(t)dt$ for $x \in [a, b]$. Then show that F and G are continuous functions of bounded variation on $[a, b]$. Also show that $f \in R(G)$, $g \in R(F)$ and $\int_a^b f(x)g(x)dx = \int_a^b f(x)dG(x) = \int_a^b g(x)dF(x)$.

   (b) State and prove the mean value theorem for Riemann- Stieltjes integrals.

OR

6. (a) If f is continuous on $[a,b]$ and α is of bounded variation on $[a,b]$, then Show that $f \in R(\alpha)$ on $[a,b]$.
   (b) Let $f \in R[a,b]$ and α is continuous on $[a,b]$ with $\alpha' \in R[a,b]$, then show that the integrals $\int_a^b f(x)d\alpha(x)$, $\int_a^b f(x)\alpha'(x)dx$ exist and are equal.

UNIT-IV

7. (a) Let one of the partial derivatives $D_1 f, \dots, D_n f$ exists at c and the remaining $n-1$ partial derivatives exist in some n-ball $B(c)$ and are continuous at c, then show that f is differentiable at c.
   (b) Find the second order Taylor expansion of $f(x,y) = e^{-(x^2+y^2)}$ about the point (1, 2).

OR

8. (a) Let u and v be two real valued functions defined on a subset S of the complex plane. Assume that u, v are differentiable at an Interior point c of S and that the partial derivatives satisfy the Cauchy-Riemann equations at c. Then show that function $f = u + iv$ has a derivative at c and $f'(c) = D_1 u(c) + iD_1 v(c)$.
   (b) Compute the directional derivative of the function $f(x,y) = x^2y^3 + 2x^4y$ at the point (1, -2) in the direction of the vector $u = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. What is the maximum value of the directional derivative.

UNIT-V

9. (a) Show that there exists a real continuous function on the real line which is no where differentiable.
   (b) Let K be a compact metric space, if $\{f_n\} \in C(K)$ for $n = 1,2,3,\dots$ and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

OR

10. (a) Define pointwise convergence and uniform convergence for a sequence of functions $\{(f_n), n = 1,2,3,\dots\}$. Test the convergence of $f_n(x) = \frac{(x+n)^2}{n^2}$, $x \in R$.
   (b) State and prove Stone-Weierstrass theorem.

M.Sc Degree Examination

First Semester

Applied Mathematics

21AM102: Ordinary Differential Equations& Integral Equations

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1. a) Let $\phi_1, \phi_2,...,\phi_n$ be n linearly independent solutions of $L(y)=y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y= 0$, on an interval I. Show that any solution φ of L(y)=0 on I, is of the form $\varphi = c_1\varphi_1 + \cdots + c_n\varphi_n$ where $c_1, \dots, c_n$ are constants.

   b) One solution of $x^3y''' - 3x^2y'' + 6xy' - 6y = 0, \forall\, x > 0$ is $\phi_1(x) = x$. Find the basis of the above differential equation $x > 0$.

OR

2. a) Let $\phi_1, \phi_2,...,\phi_n$ be n solutions of L(y)=0 on an interval I, prove that they are linearly independent if and only if $W(\phi_1, \phi_2.....\phi_n)(x) \neq 0$ for all x in I.

   b) Two solutions of $x^3y''' - 3\,xy' + 3y = 0\ (x > 0\ )$ are $\phi_1(x) = x$, $\phi_2(x) = x^3$, then find a third independent solution.

UNIT-II

3. a) Consider the equation $x^2y'' + xe^xy' + y = 0$

   i)Compute the indicial polynomial; and show that its roots are –i and i.

   ii)Compute the coefficients C_1, C_2, C_3 in the solution

   $$\phi(x) = x^i \textstyle\sum_{k=0}^{\infty} C_k x^k (x > 0)\ ,\ C_0 = 1.$$

   b) i)Show that -1 and 1 are regular singular points for the Legendre equation $(1 - x^2)y'' - 2xyy' + \alpha(\alpha + 1)y = 0$.

   ii) Find the indicial polynomial and its roots, corresponding to the point $x = 1$.

OR

4. a) Let M,N be two real valued functions which have continuous first partial derivatives on some rectangle $R: |x - x_0| \le a,\ |y - y_0| \le b$. Then prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

   b) Find an integrating factor for the following equation $(2y^3 + 2)dx + 3xy^2\, dy = 0$ and solve it.

UNIT-III

5. a) State and prove Picards existence theorem on successive approximation for the solution of I.V.P

OR

6. a) Find a solution φ of the system

$$y_1' = y_2,$$
$$y_2' = 6y_1 + y_2, \text{ satisfying } \varphi(0) = (1, -1) .$$

b) Show that the function f given by $f(x, y) = x^2|y|$ satisfies a Lipschitz condition on $R: |x| \leq 1, \; |y| \leq 1$, and find Lipschitz constant.

UNIT-IV

7. a) Obtain Fredholm integral equation of second kind corresponding to the boundary value problem $\frac{d^2\phi}{dx^2} + x\phi = 1, \;\; \phi(0) = 0, \phi(1) = 1,$ also recover the boundary value problem from the obtained integral equation.

b) Solve the integral equation $\varphi(x) = (1 + x) + \int_0^x (x - s)\varphi(s)ds$ with $\varphi_0(x) = 1$, using the method of successive approximations.

OR

8. a) Convert the differential equation $\frac{d^2\phi}{dx^2} - 2x\frac{d\phi}{dx} - 3\phi = 0$ with the initial conditions $\phi(0) = 0, \phi'(0) = 0$ to Volterra's integral equation of second kind, conversely derive the original differential equation with the initial conditions from the integral equation obtained.

b) Find the resolvent kernel of the Volterra's integral equation with the kernel $k(x, \xi) = 1$.

UNIT-V

9. a) Find the resolvent kernel of the Volterra's integral equation with the kernel $k(x, u) = \frac{2+cosx}{2+cosu}$ and there by solve the integral equation

$$\phi(x) = e^x sinx + \int_0^x \frac{2+cosx}{2+cosu} \;\; \phi(u)\, du.$$

b) Find the solution of the integral equation $\varphi(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+s^2}\varphi(s)ds$ with the help of the resolvent kernel.

OR

10. a) Find the iterated kernel for $k(x, \xi) = x - \xi$ if $a = 0, b = 1$.

b) Solve the following integral equation $\varphi(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 x\xi\varphi(\xi)d\xi$.

M.Sc. Degree Examination
First Semester
Applied Mathematics
21AM103:Classical Mechanics
(Effective from admitted batch of 2021-2022)

Time: Three hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

Unit-I

1). (a) State and explain conservation principle of angular momentum for a single particle.
(b) State and obtain Nielsen's form of the Lagrange's equations for aholonomic dynamical system.

(OR)

2). (a)State and explain D' Alembert's principle.
(b)Derive the Lagrange's equations of motion from the D' Alembert's principle.

Unit-II

3).(a)Derive the Hamilton's principle from the D' Alembert's principle.
(b)What is cyclic or ignorable coordinate. Prove that the generalized momentum conjugate to a cyclic coordinate is conserved.

(OR)

4). (a)Derive Lagrange's equations of motion from Hamilton's principle.
(b) Determine the acceleration of the two masses of a simple Atwood machine, with one fixed pulley and two masses m_1 *and* m_2.

Unit-III

5). (a) Derive the Hamilton's equations of motion from a variational principle.
(b) Obtain Hamilton's Canonical equations of motion for a simple pendulum.

(OR)

6). (a) State and proveprincipleof least action.
(b) Discuss harmonic oscillator as an example of canonical transformations.

Unit-IV

7). (a) State and prove Jacobi's Identity.
(b)Prove the invariance of Poisson brackets with respect to canonical transformation.

(OR)

8). (a) For what values of α *and* β do the equations $Q = q^{\alpha} cos(\beta p),\ P = q^{\alpha} sin(\beta p)$ represent a canonical transformation?
(b) Find the motion of one dimensional simple harmonic oscillator by Hamilton- Jacobi method.

Unit-V

9). Derive Lorentz transformation equations.

(OR)

10). (a)Explain the following:
(i) Longitudinal contraction effect.
(ii) Simultaneity.
(iii) Proper time.
(b) Show that $ds^2 = -(dx)^2 - (dy)^2 - (dz)^2 + c^2(dt)^2$is invariant under Lorentz transformation.

M.Sc Degree Examination

First Semester

Applied Mathematics

21AM104: Discrete Mathematical Structures

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1. a) Show the following implications without constructing the truth tables.
   $(P \to Q) \to Q \Rightarrow P \vee Q$.
   b) Show that the following are equivalent formulas.
   i) $P \vee (P \wedge Q) \Leftrightarrow P$.
   ii) $(P \vee \neg P \wedge Q) \Leftrightarrow P \vee Q$.

(or)

2. a) Obtain the principal disjunctive normal form of $P \to ((P \to Q) \wedge \neg(\neg Q \vee \neg P))$
   b) Obtain the principal conjunctive normal form of the formula $(\neg P \to R) \wedge (Q \leftrightarrows P)$.

UNIT-II

3. a) Demonstrate that R is a valid inference from the premises $P \to Q$, $Q \to R$, and P.
   b) Show that $S \vee R$ is tautologically implied by $(P \vee Q) \wedge (P \to R) \wedge (Q \to S)$.

(or)

4. a) Show that the following premises are inconsistent, $P \to Q$, $P \to R$, $Q \to \neg R$, P.
   b) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$.

UNIT-III

5. a) If R is a partial ordering relation on a set X and $A \subseteq X$, Show that $R \cap (A \times A)$ is a partial ordering relation on A.
   b) Let A be a given finite set and $\rho(A)$ its power set. Let $\subseteq$ be the inclusion relation on the elements of $\rho(A)$. Draw Hasse diagrams of $< \rho(A), \subseteq >$ for
   i) A={a} ii) A={a,b} iii) A={a,b,c} iv) A={a,b,c,d}

(or)

6. a) Let <L,≤> be a lattice in which $*$ and $\oplus$ denote the operation of meet and join respectively. Prove that for any a, b,$\in L, a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.
   b) Prove that every chain is a distributive lattice.

UNIT-IV

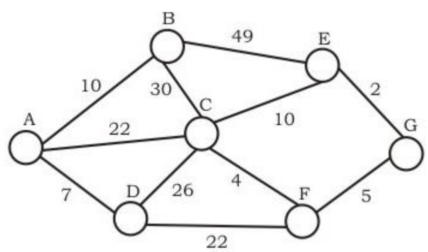
7. a) State and derive Euler's formula for graphs.
   b) Show that the number of vertices of odd degree in any graph is always even.

(or)

8. a) Define Hamiltonian and Eulerian graphs and give examples. Also give an example of a graph which is Eulerian but not Hamiltonian.
   b) Prove that a tree with n vertices has exactly (n-1) edges.

UNIT-V

9. a) Write Warshall's algorithm to find the shortest path in graphs.
   b) Find the minimal spanning tree of the following graph G and find the total weight of the minimal spanning tree by using Prim's algorithm.



(or)

10. a) Write Depth-First Search algorithm to find the spanning tree.
    b) Define a binary tree and draw the binary tree T which corresponds to the algebraic expression E=$(x+3y)^4(a-2b)$.

M.Sc Degree Examination

First Semester

Applied Mathematics

21AM105:Programming in C

(Effective from the admitted batch of 2021-2022)

Time: Three hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

Unit-I

1. (a) Discuss about operators available in C language.
   (b) Write a program in C to perform the following
      (i) Area of a circle, (ii) Circumference of a circle,
      (iii) Area of a triangle, (iv) Area of a rectangle.

(or)

2.(a)Write and explain the general forms of nested if statements.
   (b) Write a program in C to find the roots of quadratic equation using if else structure.

Unit-II

3. (a) Explain about various loop statements.
   (b) Write a C programming to check give number is palindrome or not.

(or)

4.(a) Write a program to generate prime numbers in the given range.
   (b) Write a program in C to convert given decimal number to octal number.

Unit-III

5. (a)Write a general form of the function and also write three types of functions.
   (b) Write a function to swap the values of two variables, and corresponding main program

(or)

6.(a) Explain about four different types of storage classes available in C.
   (b) Write a recursive function to compute factorial of a given integer.

Unit-IV

7.(a)Write a function to compute norm of a matrix.
   (b) Write a program in C to compute transpose of a matrix.

(or)

8.(c) Explain the following (i) Pointer variable, (ii) Pointer operator, (iii) Address operator
   (d) Write a program to copy a string to another string.

Unit-V

9. (a) Explain about call by value and call by reference and give examples.
   (b) Write a program tosort set of n numbers in ascending order using pointers.

(or)

10 (a) Explain the relation between
   (i) pointer and one dimensional array, (ii) pointers and multi dimensional arrays.
   (b) Write the general form of a structure and create a structure for students data with roll no, age, sex, height and weight and write a program to read and print the contents of the structure.