M.Sc. APPLIED MATHEMATICS (FOURTH SEMESTER)

21AM-401: FUNCTIONAL ANALYSIS
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM401

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course outcomes (COs): At the end of the course, the students will be able to

CO1: Understand the concepts of topological spaces and the basic definitions of open sets, neighbourhood, interior, exterior, closure and their axioms for defining topological space. Understand the concept of Bases and Subbases, create new topological spaces by using subspace.

CO2: Understand continuity, compactness, connectedness, homeomorphism and topological properties. Understand how points of space are separated by open sets, Housdroff spaces and their importance. Understand regular and normal spaces and some important theorems in these spaces

CO3: Explain the fundamental concepts of functional analysis and their role in modern mathematics. Utilize the concepts of functional analysis, for example continuous and bounded operators, normed spaces, Hilbert spaces and to study the behaviour of different mathematical expressions arising in science and engineering.

CO4: Understand and apply fundamental theorems from the theory of normed and Banach spaces including the Hahn-Banach theorem, the open mapping theorem, the closed graph theorem and uniform boundedness theorem.

CO5: Understand the nature of abstract mathematics and explore the concepts in further details. Explain the concept of projection on Hilbert and Banach spaces.

Learning Outcomes(LOs):

LO1: Students will be able to understand the topological-algebraical structures of the spaces.

LO2: the main properties of bounded operators between Banach and Hilbert spaces.

LO3: the basic results associated to different types of convergences in normed spaces.

LO4: the spectral theorem and some of its applications.

LO5: With this knowledge they will be able to correlate Functional Analysis to problems arising in Partial Differential Equations, Measure Theory and other branches of Mathematics.

Course Specific outcomes (CSOs):

CSO1: Learn about topological space, its base and subbase construction of new topology from old ones.

CSO2: Learn about Banach space and its properties.

CSO3: Learn about Hilbert space, operators on this space, spectral theorem.

Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	✓
CO2	✓	✓	✓	✓	✓
CO3	✓	✓	✓	✓	✓
CO4	✓	✓	✓	✓	✓
CO5	✓	✓	✓	✓	✓

Unit-I

Topological spaces: Elementary concepts, open bases and open subbases, weak topologies, function algebras Co (X, R) and Co (X, C), compact spaces, product spaces, Tychonoff's theorem, separation concepts. (Section 16-23, 26, 27 of Text book)

Unit-II

Banach spaces: Definition and some examples, continuous linear transformations, the Hahn-Banach theorem, the natural imbedding of N in N**, the open mapping theorem, the conjugate of an operator. (Chapter 9 of Text book)

Unit-III

Hilbert spaces: Definition and some simple properties, orthogonal complements, orthonormal sets, the conjugate space H* (Section 52-55 of Textbook)

Unit-IV

Operators in Hilbert space: Adjoint of an operator, Self-adjoint operators, Normal and Unitary operators, Projections. (Section 56-59 of Text book)

Unit-V

Finite-Dimensional Spectral Theory: Matrices, Determinants and the Spectrum of an operator, Spectral theorem, a survey of the situation. (Chapters 11 of Text book)

Text Book: G.F. Simmons – Mc Graw Hill, Introduction to Topology and Modern Analysis.

M.Sc. APPLIED MATHEMATICS (FOURTH SEMESTER)

21AM 402: THEORY OF AUTOMATA AND FORMAL LANGUAGES
(With effect from 2021-2022 Admitted Batches)

Course Code:21AM402

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course Outcome(COs):

CO1: The various types of finite automatons like deterministic non deterministic finite automation and conversion from DFA to NDFA and Finite automations with outputs and their classification and relations , the minimization of finite automaton will taught extensively.

CO2: Classification of formal languages and relation between automaton and languages will taught.

CO3: The regular grammar and various results on these will be discussed.

CO4: The context free grammar and various results on these will be discussed.

CO5: By introducing the memory in to FA the more powerful automaton namely Turing machine and construction of Turing machines will be taught.

Course Specific Outcome(CSOs):

CSO1: To understand and utilize the abstract automatons in developing the logics for certain computations.

CSO2: To familiar with the relations between Automatons , Grammars and Languages.

CSO3: Learn to construct most powerful automaton developed by Turing machine, Universal Turing Machine and certain properties.

Learning Outcome(LOs):

LO1: The learners are familiar with finite automaton definitions, operations and various types of finite automatons like DFA , NDFA and FA's with outputs and conversions among them and constructing FA's various computations and reducing the number of states in FA.

LO2: The learners are also familiar with formal languages and classification of grammars , to find language generated by grammar and construction of grammars to generate required language.

LO3: The various concepts in Regular grammar and to prove certain sets are not regular are well versed .

LO4: Familiar with the context free grammar and its left/right derivations ,parse trees and reducing the CFG's.

LO5: Familiar with Turing machines, operations and various types of representation of TM , operations in TM and construction of TM.

Mapping of course outcomes with the program outcomes

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	✓
CO2	✓	✓	✓	✓	✓
CO3	✓	✓	✓	✓	✓
CO4	✓	✓	✓	✓	
CO5	✓	✓	✓	✓	

Unit-I

The Theory of Automata: Definition of an automata, Description of a Finite Automation, Transition Systems, Properties of Transition Functions, Acceptability of a string by a finite Automation, Non Deterministic finite State Machines, The Equivalence of DFA and NDFA, Mealy and Moore models.

Unit-II

Minimization of Finite Automaton.

Formal Languages: Basic definitions and examples, Chomsky classification of Languages, Languages and their relation, Languages and Automaton.

Unit-III

Regular sets and Regular Grammars: regular expressions, Finite Automata and regular expressions, Pumping lemma for Regular sets, Application of Pumping lemma.

Unit-IV

Context-free Languages: Context-free languages and derivation trees, Ambiguity in Context-free Grammars, Simplification of Context-free Grammars, Normal forms for Context-free Grammars.

Unit- V

Turing Machines: Turing Machine model, Representation of Turing Machines, Languages Acceptability by Turing Machines, Design of Turing Machines, Universal Turing Machines and other modifications, Halting Problems of Turing Machines, unsolvable problems, the post correspondence problem.

Text book: Theory of Computer Science (Automata, Languages and Computation)

Chapters: 2,3,4,5.1 to 5.4 and 7.1 to7.5 ,7.9.3 By K.L.P. Mishra,

N. Chandrasekharan, PHI, Second edition

M.SC. APPLIED MATHEMATICS (FOURTH SEMESTER)
21AM 404(A): BOUNDARY VALUE PROBLEMS-II
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM404(A)

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.

Course Outcome(COs):

CO1: The qualitative properties of differential equations like local stability, various types of stability of the systems and necessary and sufficient conditions linear, weakly non linear systems and two dimensional systems are taught.
CO2: The global stability of the systems by various methods with certain applications are taught.
CO3: Certain mathematical models for one species and two species models and their local and global stability of the positive equilibrium points are taught.
CO4: Analysis and methods of non linear systems , existence of solutions , certain differential inequalities and non linear variation of parameters formula are taught.
CO5: The qualitative property oscillatory and non oscillatory properties of second order equations are taught.

Course Specific Outcome:

CSO1: Comparing the quantitative and qualitative properties of linear and weakly non-linear systems.
CSO2: Formulation of Mathematical models for population dynamics and discussing the qualitative properties of positive equilibrium points of the systems.
CSO3: Understanding the methods and analysis of non-linear systems and certain results on Oscillatory theory.
Learning Outcome:
LO1: The learner must familiar with stability, asymptotically stability and unstable concepts. The learner also familiar with various stability properties and necessary and sufficient conditions for local stability analysis.
LO2: The concepts on stability based on Liapunov second method should applied to various problems.
LO3: The mathematical modeling on single and multi species are formulated and learner are familiar with to test their local and global stability of the positive equilibrium point of the system.

LO4: With out assuming the Lipschitz condition the learner establish the existence of solutions of system of initial value problems and certain properties of the system are well versed.

LO5: Results on Oscillatory and non oscillatory properties of the differential equations are to be familiar.

Mapping of course outcomes with the program outcomes

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	
CO2	✓	✓	✓	✓	
CO3	✓	✓	✓	✓	
CO4	✓	✓	✓	✓	
CO5	✓	✓	✓	✓	

Unit-I

Stability of linear and weakly non-linear systems, continuous dependence and stability properties of linear, non-linear and weakly non-linear systems. Two dimensional systems. (chapter III of text book-1)

Unit-II

Stability by Liapunov second method, Autonomous systems, quadratic forms, Krasovski's Method. Construction of Liapunov functions for linear systems with constant coefficients. Selection of total energy function as a Liapunov Function, Stability based on first approximation (Chapter V of text book-1)

Unit-III

Mathematical Models in Population Dynamics: Introduction, single species Models, Two species Lotka volterra Models, Multi species Models. (chapter VI of text book-1)

Unit-IV

Analysis and Methods of non-linear differential equations, Existence theorem, extremal solutions, upper and lower solutions. Existence via upper and lower solutions.(Sec. 6.1-6.4 in Chapter VI of text book-2)

Bihari's inequality, Application of Bihari's integral inequality. Non-linear variation of parameters formula Alekseev's formula. (Sec. 6.1 to 6.4,6.6-6.7 in Chapter VI of text book-2)

Unit-V

Oscillations of second order equation, sturms comparison theorems, Elementary linear Oscillations, comparison theorem of Hille-Winter. (Chapter VIII of text book-2)

Text Books:

1. M.Rama Mohan Rao, Ordinary Differential equations, Theory methods and applications, Affiliated East-West Press Pvt.Ltd., New Delhi. (1980).
2. V.Lakshmikantam, S.G.Deo and V.Raghavendra, Text book of ordinary differential equations (second edition) Tata Mc Graw Hill, New Delhi. (1997).

M.Sc. APPLIED MATHEMATICS(FOURTH SEMESTER)
21AM 405(B): OPTIMIZATION TECHNIQUES-II
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM405(B)

(A total of ten questions are to be set taking two questions from each unit with internal choice in each unit. Each question carries 16 marks.)

Course Outcome(COs):

CO1: This course introduces some key concepts of optimization techniques.
CO2: Provides an in-depth knowledge of problem solving through various optimization techniques.
CO3: Tests and develops the students' knowledge of basic understanding of the problems through practical illustrations and examples.
CO4: Improves the logical thinking ability of the students and helps gain access to various employment opportunities.
CO5: Infuses practical knowledge that helps in pursuing higher studies as well as getting employment.

Course Specific Outcome(CSOs):

CSO1: Study on some optimization techniques like Game theory, job sequencing techniques, exposure to inventory management and Replacement problems, their solution techniques in real world problems.
CSO2: Introducing the concepts of Queueing models and their solutions.
CSO3: Gain knowledge of finding critical paths in network analysis. These concepts of optimization have been further expanded to study Network analysis and scheduling techniques.

Learning Outcome(LOs):

LO1: After studying the students are expected to learn some techniques of optimization.
LO2: Gain ability to problem formulations, analysis and solution techniques.
LO3: Improves the logical thinking ability.
LO4: Provides an in-depth knowledge of problem solving through various optimization techniques.
LO5: Gain knowledge in computational skills and numerical problem solving.

Mapping of course outcomes with the program outcomes

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓		✓	
CO2			✓		
CO3		✓	✓		
CO4	✓			✓	✓
CO5		✓		✓	✓

Unit-I

Game Theory, Solution of Games with and without saddle points, minimax / maximini principle, principle of Dominance, matrix method for (m X n) Games without saddle point, algebraic method.**(Chapter 1 of Unit 4(except 1.22)**
Jog Sequencing: Processing of n-jobs through 2/3/m$^{machines(}$**Chapter 6 of unit 4)**

Unit-II

Inventory, classification inventory models, EOQ models with and without shortages, multi item deterministic models, dynamic demand Models. **(Chapter 2 of unit 4 (2.1 to 2.17))**

Unit-III

Replacement Models: Replacement of items that deteriates with time, individual replacement. Group replacement policies, recruitment and production problem.Equipment and renewal problem systems reliability.**(Chapter 4 of unit 4)**

Unit-IV

Queuing theory: distribution in queuing systems, Poisson process. Classification and solutions of Queuing model, models 1-4 **(Chapter 5 of unit 4) (5.1 to 5.15)**

Unit-V

Network analysis, PERT/ CPM Techniques network diagram representation time estimates and critical path in net work analysis, uses of PERT / CPM Techniques **(Chapter7 of unit 4)**

Text book: Operations Research by S.D.Sarma (12 th Edition), KedarnathRamnath and company.

M.Sc. APPLIED MATHEMATICS (FOURTH SEMESTER)
21AM 406(C):RELATIVITY AND COSMOLOGY-II
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM406(C)

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course Outcome(COs): At the end of the course, the students will be able to

CO1: Describe the basic concepts of the theory of relativity.
CO2: Explain the concept of invariance.
CO3: Explain the concept of space-time.
CO4: Discuss the equivalence principle and explain the twin paradox.
CO5: Describe gravity as space-time curvature. Also, describe general theory of relativity as mathematical basis of physical cosmology.

Course Specific Outcomes(CSOs):

CSO1: The students know how Einstein used the Equivalence Principle to extract some general relativity results (gravitational redshift, time dilation, and light deflection) and in turn how such physics led him to the idea that gravitational field is simply a curved spacetime.
CSO2: The students gain knowledge of the different cosmological parameters for understanding the observed universe.
CSO3: The students learn many features of the universe which are not understood by Special Theory of Relativity and Classical Mechanics.

Learning Outcomes(LOs): After completing the course, you:

LO1: master the equivalence principle and have a good knowledge of how this leads to a geometric description of gravity, in the form of the general theory of gravity.
LO2: have detailed knowledge about how space and time are curved for spherically symmetric mass distributions**.**
LO3: have acquired basic knowledge about the cosmological concordance model, and how it is based on Einstein's theory of gravity.
LO4: are able to present complex topics in general relativity in a clear and pedagogic way, and communicate this to fellow students.
LO5: can communicate the basic principles behind the theory, as well as tests of the theory, to people outside the community.

Mapping of course outcomes with the program outcomes					
	PO1	**PO2**	**PO3**	**PO4**	**PO5**
CO1	√	-	√	√	-
CO2	√	-	√	√	-
CO3	√	-	√	√	-
CO4	√	-	√	√	-
CO5	√	-	√	√	-

Unit-I

The Fundamental Principles of General Relativity, Principle of Covariance, Principle of equivalence, Principle of Mach, Gravitational field in empty space, Gravitational field in the presence of matter and energy. Simple consequences of principle of equivalence, Newton's theory as a first approximation. The Schwarzschild line element, the three crucial tests of Relativity(Chapter VII-Articles 94-99 of Text book).

Unit-II

Line elements for systems with spherical symmetry, static line element with spherical symmetry, Schwarzschild exterior and interior solutions, Non-static line elements with spherical symmetry-Birkhoff's theorem. The generalized Lorentz Electron theory the field equations. The gravitational field of a charged particle (Chapter VII-Articles 102 &107 of Text book).

Unit-III

The Energy-Momentum Tensor for Disordered Radiation. The Gravitational Mass of Disordered Radiation. The Energy-Momentum Tensor Corresponding to a Directed Flow of Radiation. The Gravitational Field Corresponding to a Directed Flow of Radiation. The Gravitational Action of a Pencil of Light. The Gravitational Action of a Pulse of Light. Discussion of the Gravitational Interaction of Light Rays and Particles. The Generalized Doppler Effect(Chapter VIII-Articles 120 &127 of Text book).

Unit-IV

Application of general relativity to cosmology, The three possibilities for a homogeneous static universe, The Einstein line element, the de-sitter line element, Special relativity line element, The geometry of the Einstein universe, Density and pressure of material in Einstein universe. Behavior of test particles and light rays in the Einstein universe (Chapter X-Articles 133-139 of Text book).

Unit-V

Comparison of Einstein model with actual universe, Geometry of the de-sitter universe, Absence of matter and radiation from de-sitter universe, Behavior of test particles and light rays in the de-sitter universe(Chapter X-Articles 140-144 of Text book).

Text Book:

Relativity, Thermodynamics and Cosmology, R.C. Tolman, Clarendon Press, Oxford.

Books for reference:

1. Theory of Relativity by S.R. Roy and Raj Bali, Jaipur Publishing House, Jaipur.

M.Sc. APPLIED MATHEMATICS (FOURTH SEMESTER)
21AM 407(D): Numerical solution of Partial Differential Equations-II
(With effect from 2021-2022 Admitted Batch)
Course Code:21AM407(D)

(A total of Ten questions to be set by selecting two questions from each unit with internal choice. Each question carries 16 marks.)

Course Outcomes(COs):

On completion of this course, the students will have the ability to

CO1 :Understand basic theory, importance, efficiency of finite element methods.

C02: solve elliptic partial differential equations using weak formulations and finite element Methods.

C03 :construct various finite elements to approximate the solution of PDEs.

C04 :analyses the convergence analysis to identify the robustness and possible improvement of the finite element methods.

C05: implement finite element formulation of various partial differential equations.

Course Specific outcomes (CSOs):

CSO1: Learn about solution of ODEs numerically stable schemes.

CSO2: Learn about Parabolic PDEs and finding its solution numerically.

CSO3: Learn about Elliptic PDEs and finding its solution numerically

Learning outcome(LOs):

LO1: At the end of the course, a student will be able to explain the concepts and principles used in the formulation and application of the finite element method.

LO2: demonstrate an ability to formulate, implement, and document solutions to solve simple engineering problems using the finite element method.

LO3: Learn about the function spaces.

LO4: Interpolation and polynomial approximation in finite element spaces.

LO5: They will be able to construct stable finite element methods for solving problems in science, engineering, option pricing.

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Mapping of course outcomes with the program outcomes:

	PO1	PO2	PO3	PO4	PO5
CO1	✓	✓	✓	✓	✓
CO2	✓	✓	✓	✓	✓
CO3	✓	✓	✓	✓	✓
CO4	✓	✓	✓	✓	✓
CO5	✓	✓	✓	✓	✓

Unit-I

Test functions and Distributions, operations with distributions, support of distributions, convolution of distributions, definition and basic properties. (Section 1.1, 1.2, 1.3, 1.5, 1.6 of Text book 1).

Unit-II

Variational problem, variational formulation of one-dimensional model problem, FEM for the model problem with piecewise linear functions, Ritz-Galerkin method, Error estimate for the model problem, FEM for the Poisson equation, Geometrical interpretation of FEM, Hilbert spaces, A Neumann problem: Natural and essential boundary conditions. (Section 1.1 to 1.7 of Textbook 2)

Unit-III

Abstract formulation of FEM for Elliptic problems: The continuous problem, Lax-Milgram theorem, stability estimate, discretization, error estimates, energy norm. (Chapter 2, of Textbook 2)

Unit-IV

Finite element spaces: Regularity requirements, Examples of finite elements. Approximation theory for FEM. Error estimates for elliptic problems: introduction, interpolation with piecewise linear functions in two dimensions, error estimates for FEM for elliptic problems, Regularity of exact solution, Error estimate in L2 norm, (chapter 3, 4 of Textbook 2)

Unit-V

FEM for Parabolic Problems, One Dimensional model problem, semi discrete and fully discrete Scheme, Error estimates. (Chapter 8 of Textbook 2)

Text Books:

1. S Kesavan, Topics in functional analysis and applications, new age international limited, 2003.
2. Claes Johnson, Numerical solutions of partial differential equations by the finite element method, Cambridge university press, Cambridge, 1987.

Reference Books:

1. S.C Brenner, R. Scott, The mathematical theory of finite element methods, Springer, New York, 2008.
2. J.N. Reddy, An Introduction to the Finite Element Method, Tata McGraw-Hill Edition.