M.Sc Degree Examination

Second Semester

Applied Mathematics

21AM201: Complex Analysis

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT – I

1). (a) Let $f(z) = f(x+iy) = u(x,y) + iv(x,y)$be differentiable at the point $z_0 = x_0 + y_0$.Then prove that the partial derivatives of u and v exist at the point(x_0, y_0) and satisfy the equations $u_x(x_0,y_0) = v_y(x_0,y_0)$ and$u_y(x_0,y_0) = -v_x(x_0,y_0)$.

(b) Let f be an analytic function in the domain D. If $|f(z)|$=k, where k is a constant, then prove that f is constant in D.

(OR)

2). (a) Let $f(z) = u(x,y) + iv(x,y)$ be an analytic function in the domain D. If all second order partial derivatives of u and v are continuous then prove that u and v are harmonic functions in D.

(b) Show that $u(x,y) = xy^3 - x^3y$ is a harmonic function and find the conjugate harmonic function $v(x,y)$.

UNIT – II

3).(a) State and prove Cauchy- Goursat Theorem.

(b) State and prove Liouville's theorem.

(OR)

4) (a) State and prove Cauchy's integral formula.

(b) Evaluate the following integral $\int_c (z - 3/z^2 + 2z + 5)dz$,where C is the circle $|z| = 1$.

UNIT –III

5) State and prove Laurent's theorem.

(OR)

6) (a) Obtain two Laurent series expansion in powers of $z$for the function $f(z) = \frac{1}{z^2(1-z)}$ and specify the region in which those expansions are valid.

(b) Locate the singularities of the following functions and determine their type:

(i) $z \exp\left(\frac{1}{z}\right)$ (ii) $\frac{z^2}{z-sin}$

UNIT-IV

7). (a) State and prove Cauchy's residue theorem.

(b) Using the theory of residues, evaluate $\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)} dx$

(OR)

8). (a) State and prove Rouche's theorem.

(b) Let $g(z) = z^5 + 4z - 15$

(i). Show that there are no zeros in $|z| < 1$.

(ii). Show that there are five zeros in $|z| < 2$.

UNIT -V

9). (a) Find the image of the upper half plane $Im(z) > 0$ under the transformation $w = \frac{(1-i)z+2}{(1+i)z+2}$.

(b) Find the bilinear transformation $w = s(z)$ that maps the points $z_1 = 0, z_2 = i \text{ and } z_3 = -i \text{ onto } w_1 = -1, w_2 = 1 \text{ and } w_3 = 0$ respectively.

(OR)

10). (a) Find the fixed points of

(i). $w = \frac{z-1}{z+1}$ (ii). $w = \frac{4z+3}{2z-1}$

(b) Show that $\boldsymbol{w} = \boldsymbol{s}(\boldsymbol{z}) = \frac{\boldsymbol{i}(\mathbf{1}-\boldsymbol{z})}{\mathbf{1}+\boldsymbol{z}}$ maps the unit disk $|\boldsymbol{z}| < 1$ one-to-one and onto the upper half plane $Im(z) > 0$.

M.Sc Degree Examination

Second Semester

Applied Mathematics

21AM202: Partial Differential Equations & Integral Transforms

(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-1

1. (a) Find the integral curves for the sets of equations
$\frac{dx}{xz-} = \frac{dy}{yz-} = \frac{dz}{1-z^2}$.

   (b) Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 tan^2 \alpha$ of its intersections, with the family of planes parallel to $z = 0$.

(OR)

2. (a) Prove that the pfaffian differential equation $\bar{X}.d\bar{r} = 0$ is integrable if and only if $\bar{X}.curl\bar{X} = 0$.

   (b) Find the complete integrals of the equation $(p^2 + q^2)y = qz$.

UNIT-II

3. (a) Solve the equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^4 z}{\partial x^2 \partial y^2}$.

   (b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

(OR)

4. (a) Solve the wave equation $r = t$ by Monge's method.

   (b) Solve the equation $r + 4s + t + rt - s^2 = 2$.

UNIT-III

5. (a)A string is stretched and fastened to two points at a distance l apart. Motion is started by displacing the string in the form $y = a \sin\frac{n\pi}{l}$, from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by $y(x,t) = a \sin\frac{\pi x}{l} \cos\frac{\pi ct}{l}$.

OR

6. (a) A homogeneous rod of conducting material of length 100cm has its ends kept at zero temperature and the temperature initially is $u(x,0) = x,\ 0 \le x \le 50$
$u(x,0) = 100 - x,\ 50 \le x \le 100$. Find the temperature $u(x,t)$ at any time.

UNIT-IV

7. (a) Solve $(D^3 - 3D^2 + 3D - 1)y = t^2e^t, y(0) = 1, y'(0) = 0, y''(0) = -2$

by using Laplace transform.

(b) A particle moves along the line so that its displacement *X*, from a fixed point at any time t is given by $X''(t) + 4X'(t) + 5X(t) = 80\sin 5t$. Find its displacement at any time $t > 0$, if at $t = 0$ the particle is at rest $X = 0$.

(OR)

8. (a) Using convolution theorem, show that $\int_0^t \sin u \cos(t-u)du = \frac{t}{2}\sin t$.

(b) Using Laplace transform solve $\frac{\partial^2 y}{\partial x^2} + y = t\cos 2t$. with $y = 0, y' = 0$ when $t = 0$.

UNIT – V

9. (a) Find the Fourier transform of *f(x),* if $f(x) = \begin{cases} \frac{1}{2\varepsilon}, & |x| \le \varepsilon \\ 0, & |x| > \varepsilon \end{cases}$.

(b) State and prove Fourier integral theorem.

(OR)

10. (a) Find the Fourier cosine transform of $f(x) = \frac{1}{1+x^2}$ and hence derive Fourier sine transform of $\emptyset(x) = \frac{x}{1+x^2}$.

(b) Solve the one- dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$, using the Fourier transform to determine the displacement $y(x,t)$ of an infinite string, given that the string is initially at rest.

M.Sc Degree Examination
Second Semester
Applied Mathematics
21AM203: Statistics & Distribution Theory
(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1) a) The distribution function of a random variable X is given by

$$F(x) = \begin{cases} 1 - K(1+x)e^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Find the value of K and the corresponding density function of X.

b) Define mathematical expectation and explain the additive and multiplicative properties of expectation of two random variables.

OR

2) a)State and prove Chebychev's inequality.

b) Let $(X, Y\ )$ be jointly distributed with pdf

$$f(x,y) = \begin{cases} 2, & 0 < x < y < 1 \\ 0, & otherwise \end{cases}$$

Find the marginal and conditional pdfs of $X$and Y. Also compute $P(Y \geq \frac{1}{2} | X = \frac{1}{2})$ and $P(X \geq \frac{1}{3} | Y = \frac{2}{3})$.

UNIT–II

3) a)Derive the moment generating function of binomial distribution and explain the additive property of binomial random variate.

b) Define gamma and beta distributions and establish the relationship, if any, between the corresponding random variables.

OR

4) a) Explain the uses of normal distribution. Also show that the mean deviation about the mean for normal distribution is $\frac{4}{5}\sigma$ approximately.

b) The number of customers arriving per hour at a certain automobile service facility is assumed to follow a Poisson distribution with mean $\lambda = 7$.

(i) Compute the probability that more than 10 customers will arrive in a 2-hour period.

(ii) What is the mean number of arrivals during a 2-hour period?

UNIT –III

5) a) Find the angle between the two regression lines and explain the cases when (i) the two variables are uncorrelated (ii) variables are perfectly correlated.

b) The table below shows the IQ of 7 fathers and their eldest sons:

IQ of father	91	97	102	103	105	110	114
IQ of son	102	94	105	115	113	99	116

Calculate the correlation coefficient between the IQ of father and son and comment briefly whether this value supports the theory that IQ is an inherited factor.

OR

6) a) From the following information, calculate the regression equations: $\sum x = 30; \sum y = 40; \sum xy = 214; \sum x^2 = 220; \sum y^2 = 340; N = 5$. Also find the coefficient of correlation.

b) Derive rank correlation coefficient and find its limits.

UNIT –IV

7) a) Explain the steps involved in testing a hypothesis.

b) A machine puts 9 imperfect articles in a sample of 200. After the machine is overhauled, it puts out 4 imperfect articles in a batch of 100. Test at 5% level of significance whether the machine has been improved after overhauling.

OR

8) a) In a sample of 300 units of manufactured products, 65 units were found to be defective and in another sample of 200 units there were 35 defectives. Is there any significant difference in the proportion of defectives in the samples at 5% level of significance?

b) In a test given to two groups of students, the marks obtained are as follows:

First group	19	22	23	45	50	35	56	44	39
Second Group	27	55	34	24	33	44	26	40	27

Examine the significance of the difference between the arithmetic mean of the marks secured by the two groups of the students. Test at 5% level of significance.

UNIT –V

9) a) Derive Student's t-distribution and find its variance.

b)A random sample of 220 students in a college were asked to give opinion in terms of Yes or No about the winning of their college team in a tournament. The following data is collected:

	Class in College		
	I year	II year	III year
Yes	43	20	37
No	23	57	40

Test whether there is any association between opinion and class in college using 5% level of significance?

OR

10) a) If X_1 and X_2 are two independent χ^2 variates with n_1 and n_2 degrees of freedom respectively, then prove that $\frac{X_1}{X_2}$ is a $\beta_2\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$ variate.

b) Explain F distribution and derive the relation between t and F distribution.

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M.Sc Degree Examination
Second Semester
Applied Mathematics
21AM204: Elements of Elasticity & Fluid Dynamics
(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1. (a) Derive the equation of continuity for an incompressible fluid.

(b)Explain general analysis of fluid motion.

(OR)

2 (a) Discuss the acceleration of a fluid.

(b) Test whether the motion specified by $\bar{q} = \frac{k^2\,(x\bar{j}-y\bar{\imath})}{x^2+y^2}$ where k is a non zero is a possible form of motion for an incompressible fluid, if so find its vorticity vector?

UNIT-II

3. (a) Define the circulation. State and prove Kelvin's circulation theorem.

b) Write about pitot tube and venturi tube.

(OR)

4. (a) With usual notation derive the Bernoulli's equation for inviscid irrotational fluid motion.

(b) Derive Euler’s equation of motion in vector form and write its cartesian form.

UNIT-III

5. (a) Discuss the steady uniform flow past a fixed long infinite circular cylinder.

(b) Describe the irrotational motion of an incompressible liquid for which the complex potential is $i\kappa \log z$.

(or)

6.a) State and prove the Milne-Thomson circle theorem.

b) Using this theorem discuss the steady uniform flow past a stationary circular cylinder

UNIT-IV

7. (a) Define an affine transformation and show that an affine transformation carries straight line segments into straight line segments

   (b) Explain about principle strains and invariants.

(or)

8. (a) Write about geometrical interpretation of strain components e_{23}.

   (b) Derive the equations of compatibility in terms of strain components.

UNIT-V

9. (a) Define a stress tensor and show that it is symmetric.

   (b) Derive equations of equilibrium.

(or)

10. (a) Explain in detail about Mohr's circle.

    (b) Prove that the expression $T_{11}+T_{22}+T_{33}$ is invariant under an orthogonal transformation of coordinates.

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M.Sc Degree Examination
Second Semester
Applied Mathematics
21AM205: Numerical Analysis
(Effective from the admitted batch of 2021-2022)

Time: 3 hours | Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1. (a) Using Chebyshev method, find the root of the equation $f(x) = \cos x - xe^x = 0$ correct to six decimal places.
   (b) Derive the rate of convergence of Newton-Raphson Method.

OR

2. (a) Perform three iterations of the Bisection method to find the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$.
   (b) Derive the rate of convergence of Regula-Falsi Method.

UNIT-II

3. (a) Find the inverse of the coefficient matrix of the system

$$x_1 + x_2 + x_3 = 1$$
$$4x_1 + 3x_2 - x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

   by Gauss-Jordan Method.

   (b) Solve the system of equations

$$x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 + 8x_2 + 22x_3 = 6$$
$$3x_1 + 22x_2 + 82x_3 = -10$$

   by Cholesky Method.

OR

4. (a) Solve the system of equations

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_3 = -4$$

   using Jacobi iteration method and its residual approach. Taking the initial approximation $x^{(0)} = (0.5, -0.5, -0.5)^T$, perform three iterations in each case.

   (b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{bmatrix}$ using Jacobi Method.

UNIT-III

5. (a) Construct the Hermite interpolating polynomial that fits the data

x	0.0	0.5	1.0
$f(x)$	0	0.4794	0.8415
$f'(x)$	1	0.8776	0.5403

Estimate the value of $f(0.75)$.

(b) Find the approximate value of the integral $I = \int_0^1 \frac{dx}{1+x}$ using composite Simpson's 1/3rd rule with 3,5,9 nodes and Romberg integration.

OR

6. (a) Determine the least square approximation of the type $ax^2 + bx + c$ to The function 2^x at the points $x_i = 0,1,2,3,4$.

(b) Fit the following four points by cubic splines:

X	1	2	3	4
y	1	5	11	8

use the end conditions $y''(1) = 0 = y''(4)$. Hence compute $y(1.5)$ and $y''(2.5)$.

UNIT-IV

7. (a) Solve the IVP $u' = t^2 - u^2, u(0) = 1, t \in [0, 0.6]$ using 3rd order Adam's Bashforth method with $h = 0.1$. Take the starting value using 3rd order Taylor series Method.

(b) Solve the IVP $u' = \sqrt{t+u}$, $u(0.4) = 0.41$, $t \in [0.4, 0.8]$ with $h = 0.2$ using classical Runge-Kutta 4th order Method.

OR

8. (a) Solve the IVP $u' = -u^2, u(1) = 1$ using Euler Method. Compute $u(1.2)$ with $h = 0.1$.

(b) Solve the IVP $u' = -2tu^2, u(0) = 1$ with $h = 0.2$ on [0, 0.4] using the $P - C$ Method.

$$P: u_{j+1} = u_j + \frac{h}{2}(3u_j' - u_{j-1}')$$

$$C: u_{j+1} = u_j + \frac{h}{2}(3u_{j+1}' + u_j') \text{ as } P(EC)^2E.$$

UNIT-V

9. (a) Find the characteristics of the following equation and reduce it to the appropriate canonical form $u_{xx} - 4u_{xy} + 4u_{yy} = \cos(2x + y)$.

(b) Find the solution of $u_{xx} + u_{yy} = 0$ in R subject to the condition $u(x, y) = x - y$ on the boundary ∂R, where R is the region inside the triangle with vertices (0,0), (7,0), (0,7) using five point formula. assuming uniform step length $h = 0.2$ along the axes.

OR

10.(a) Classify the following PDE and reduce it to its appropriate canonical form

$$u_{xx} - xu_{yy} = 0 .$$

(b) Find the solution of $u_{xx} + u_{yy} = x + y$ in R subject to the condition $u(x, y) = \frac{x^2+y^2}{2}$ on the boundary ∂R, where R is a triangle $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq x + y \leq 1$ using five point formula. Assuming uniform step length $h = \frac{1}{4}$ along the axes.