M.Sc Degree Examination
Third Semester
Applied Mathematics
21AM301: Measure Theory
(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks.

UNIT-I

1. (a) Prove that the outer measure of an interval is it's length.
   (b)Let f be an extended real valued function whose domain D is measurable . Then prove that the following are equivalent
   (i) For each real number α the set$\{ x: f(x) > \alpha\}$ is measurable.
   (ii) For each real number α the set$\{ x: f(x) \geq \alpha\}$ is measurable.

OR

2. (a) If f is a measurable function and $f = g$ a.e, then show that g is measurable.
   (b) Let (E_n) be an infinite decreasing sequence of measurable sets with $E_{n+1} \subset E_n$ for each n. Let $m(E_1) < \infty$, Then prove that $m(\bigcap_{i=1}^{\infty} E_i) = \lim_{n\to\infty} m(E_n)$.

UNIT-II

3. (a)State and prove bounded convergence theorem for sequence of measurable functions.
   (b) If $\phi \; and \; \Psi$ are simple functions which vanish outside a set of finite measure , then show that $\int(\alpha\phi + \beta\Psi) = \alpha\int\phi + \beta\int\Psi$, where α, β are constants.

OR

4. (a) Let f be a bounded measurable function defined on a set E of finite measure. Let A and B are disjoint measurable sets of finite measure, then prove that

$$\int_{A\cup B} f = \int_A f + \int_B f$$

   (b) State and prove Lebesgue convergence theorem.

UNIT-III

5. (a)State and prove Vitali covering lemma.

   (b) Let f and g be non-negative and continuous at c. Then show that
   $D^{+}(fg)(c) \leq f(c)D^{+}g(c) + g(c)D^{+}f(c)$.

OR

6. (a)Prove that a function f is of bounded variation on [a, b] iff f is the difference of two monotone real valued functions on [a, b].

   (b) Let $f: [a, b] \to R$ and $g: [a, b] \to R$ and $x \in [a, b]$. Then show that
   $V_a^x(f+g) \leq V_a^x(f) + V_a^x(g)$ and $V_a^x(cf) = |c|V_a^x(f)$ for some constant $c \in R$.

UNIT-IV

7. (a) Let f be an increasing real valued function on [a, b]. Then show that f is differentiable almost everywhere and also show that f' is measurable and
   $\int_a^b f'(x)dx \leq f(b) - f(a)$.

   (b) Let f be integrable on $[a, b]$ and $\int_a^x f(t)dt = 0 \; for \; all \; x \in [a, b]$, then show that $f(t) = 0 \; a.e \; in \; [a, b]$.

OR

8. (a) State and prove Jensen's Inequality.

   (b) If f is absolutely continuous on $[a, b]$and $f'(x) = 0 \; a.e$, then show that f is constant.

UNIT-V

9. (a) State and prove Riesz- representation theorem for bounded linear functionals.

   (b) If $f \in L^p$ and $g \in L^q$, then prove that $f.g \in L^1$and $\int |fg| \leq ||f||_p.||g||_q$, where p, q are non negative extended real numbers such that $\frac{1}{p} + \frac{1}{q} = 1$.

OR

10. (a) Prove that L^p spaces are complete , $1 \leq p < \infty$.

    (b)Let g be an integrable function on [0, 1] and $|\int fg| \leq M||f||_p$ for some constant M and all bounded measurable functions f. Then prove that $g \in L^q$.

M.Sc Degree Examination
Third Semester
Applied Mathematics
21AM302: Python Programming
(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT- I

1) a) Give a brief explanation of the history of Python.
   b) Explain type conversion in python with examples.

OR

2) a)Discuss about the operators used in Python language.
   b) Write a program to convert temperature from Centigrade (read it as float value) to Fahrenheit.

UNIT-II

3) a)Differentiate the syntax of if...else and if...elif...else with an example.
   b) Write a program to check whether a number is prime or not using if blocks.

OR

4) a)Explain about *args and **kwargs. Write a python program to demonstrate their use.
   b) Write a program to find the largest of three numbers using functions.

UNIT-III

5) a) Explain the basic string operations in Python with examples.
   b)Write Python program to count the total number of vowels, consonants and blanks in a string.

OR

6) a) With the help of an example explain the concept of nested lists. Explain the ways of indexing and slicing the list with examples.
   b) Write Python program to add two matrices.

UNIT-IV

7) a)Define a dictionary. What are the advantages of using dictionary over lists.
   b)Write a Python program to input information for n number of students as given below:
   I. Name
   II. Registration Number
   III. Total Marks
   The user has to specify a value for n number of students. The program should output the registration number and marks of a specified student given his name.
   .

OR

8) a) Explain the relation between Tuples and Dictionaries.
   b) Write Python program to swap two numbers without using intermediate/temporary variables.

UNIT-V

9) a) Explain the different file mode operations with examples.
   b)Explain the ways to read and write binary files.

OR

10) a)Examine the different types of inheritances with an example
    b) Given a point(x, y), write Python program to find whether itlies in the first, second, third or fourth quadrant of $x - y$ plane.

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M.Sc Degree Examination
Third Semester
Applied Mathematics
21AM303:Techniques of Applied Mathematics
(Effective from the admitted batch of 2021-2022)

Time: 3 hours   Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT-I

1.(a) Find the solution of the difference equation,

$$x(n+3)+3x(n+2)-4x(n+1)-12x(n)=0.$$

(b) Show that the operators Δ and Δ^{-1} are linear.

OR

2. (a)(i) If $p(E)=a_0E^k+a_1E^{k-1}+\cdots+a_kI$; where E is the shift operator and $g(n)$ is a discrete function, prove that $(E)\big(b^n g(n)\big)=b^n p(bE)g(n)$.

(ii) For fixed $k\in\mathbb{Z}^+, x\in\mathbb{R}$, prove that $\Delta x^{(k)}=kx^{(k-1)}$, where Δ is forward difference operator.

(b)State and prove Abel's lemma for difference equations.

UNIT-II

3.(a)Solve the initial value problem $y(n+2)-5y(n+1)+6y(n)=2^n, y(1)=y(2)=0$ by the method of variation of parameters.

(b) Solve the difference equation $x(n+1)=\frac{2x(n)+3}{3x(n)+2}$.

OR

4. (a) Explain Pielou logistic equation and derive it's solution.

(b) Find the conditions under which the solutions of the equation

$$y(n+2)-\alpha(1+\beta)y(n+1)+\alpha\beta y(n)=1, \alpha,\beta>0.$$

(i) Converge to the equilibrium point y^*, and

(ii) Oscillate about y^*.

UNIT-III

5. (a) Find A^n if A is given by $\begin{bmatrix}0 & 1 & 1\\ -2 & 3 & 1\\ -3 & 1 & 4\end{bmatrix}$.

(b) Find the general solution of the system of difference equations $x_1(n+1)=-x_1(n)+x_2(n)$; $x_2(n+1)=2x_2(n)$ with $x_1(0)=1$; $x_2(0)=2$ by discrete Putzer algorithm.

OR

6. (a) Prove that for every fundamental matrix $\phi(n)$ of the system $x(n+1) = A(n)x(n)$, there exists a non-singular periodic matrix $P(n)$ of period N such that $\phi(n) = P(n)B^n$.

(b) Prove that the particular solution of $y(n+1) = A(n)y(n) + g(n)$, $y_p(n_0) = 0$ is $y_p(n) = \sum_{r=n_0}^{n-1} \phi(n, r+1)g(r)$.

UNIT-IV

7. (a) Solve the difference equation $x(n + 2) + 3x(n + 1) + 2x(n) = 0$ with $x(0) = 1, x(1) = -4$ by Z-transform method.

(b) Find the Z transform and its radius of convergence of the sequence $g(n) = a^n \cos(\omega n)$.

OR

8. (a) Find the inverse Z-transform of $\tilde{x}(z) = \frac{z(z-1)}{(z-2)^2(z+3)}$.

(b) Let R be the radius of convergence of $\tilde{x}(z)$. Show that

$$z\,[n^k x(n)]=(-z\frac{d}{dz})^k\, \tilde{x}(z) \text{ for } |z|>R.$$

UNIT-V

9. (a)Derive Euler's equation for minimizing the functional $v[y(x)] = \int_{x_0}^{x} f(x, y, y')\, dx$ where $y(x_0) = y_0$, $y(x_1) = y_1$.

b)On what curves can the functional $v[y(x)] = \int_0^1 [y'^2 + 12xy]dx, y(0) = 0, y(1) = 1$ be extremized.

OR

10. (a) Find the curve connecting given points A and B which is traversed by a particle sliding from A to B in the shortest time .

(b) Find the extremals of the functional

$$v[y(x)] = \int_{x_0}^{x_1} \left[y''^2 - 2y'^2 + y^2 - 2y\sin x\right] dx.$$

M.Sc Degree Examination

Third Semester

Applied Mathematics

21AM304(A):Boundary Value problems-I

(Effective from the admitted batch of 2021-2022)

Time: Three hours Maximum: 80 marks

Answer one question from each unit

All questions carry equal marks

Unit-I

1. (a) Prove that the solutions of IVP depends continuously on initial conditions.

   (b) Find the solution of $y' = \begin{pmatrix} 0 & 1 \\ -\lambda^2 & 0 \end{pmatrix} \mathrm{y},\ \mathrm{y}(\tau) = \bar{b}.$

(or)

2. (a) State and prove existence theorem for the solution of system of equations

$$y' = \mathrm{f(x, y)}, \qquad \mathrm{y}(t_0) = y_0.$$

   (b) Obtain a sequence of vectors which converges to solution for the IVP

$$y' = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \mathrm{y},\ \mathrm{y}(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

Unit-II

3. (a) Establish the relationship between the solution of scalar and vector adjoints.

   (b) Find the particular solution of the equation system $y' = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} y + \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$

(or)

4.(a) If u is any vector whose components are linearly independent functions in $c^n[a, b]$, then the differential equation for which u is a fundamental vector is $Ly = 0$, where $Ly = y^n - \widetilde{d_n}\ k'(u)k^{-1}(u)k(y).$

   (b) Show that $p_0 u'' + p_1 u' + p_2 u = 0$ is self-adjoint if $p_1 = p_0'.$

Unit-III

5. (a) Find a fundamental matrix for $y' = \mathrm{Ay}$ where $A = \begin{pmatrix} 1 & 3 & 8 \\ -2 & 2 & 1 \\ -3 & 0 & 5 \end{pmatrix}.$

   (b) Find the index of compatibility and solution space of the boundary value problem

$$y' = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \mathrm{y}, \qquad \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathrm{y}(0) + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \mathrm{y}(1) = 0.$$

(or)

6.(a) Determine the fundamental matrix for

$$y' = \begin{pmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{pmatrix} y + \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

(b) With usual notation find the adjoint boundary value problem for $y' = Ay$,

$$w(a)y(a) + w(b)y(b) = 0.$$

Unit-IV

7. (a) Find a formula involving Green's matrix for the solution of $y' = Ay + f$,

$$w^{[a]}y(a) + w^{[b]}y(b) = 0.$$

(b) State and prove the properties of the above Green's matrix.

(or)

8. (a) Find the Green's function to the boundary value problem

$$u'' = 0,$$
$$u(0) - u'(0) = 0,$$
$$u(1) + u'(1) = 0.$$

(b) Find the values of the parameter λ for which the boundary value problem $u'' + \lambda^2 u = 0, u(0) = 0, u(\pi) = 0$ is compatible.

Unit-V

9. (a) Prove that the constant system $\dot{x} = Ax + Bu$ is completely controllable if and only if the $n \times nm$

controllability matrix $U = [B, AB,\ A^2B, \dots, A^{n-1}B]$ has rank n.

(b) Show that the system $\dot{x} = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$ is completely controllable.

(or)

10 (a) Prove that the system $\dot{x} = A(t)x(t) + B(t)u(t),\ y = C(t)x(t)$ is completely observable if the

symmetric observability matrix

$$V(t_0, t_1) = \int_0^t \Phi^T(\tau, t_0) C^T(\tau) C(\tau) \Phi(\tau, t_0)\, d\tau$$

is non singular.

(b) Show that system $\dot{x}_1 = a_1x_1 + b_1u,\ \dot{x}_2 = a_2x_2 + b_2u,\ y = x_1$ is completely observable.

M.Sc Degree Examination
Third Semester
Applied Mathematics
21AM305(B): Optimization Techniques-I
(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

UNIT- I

1) a) Solve the LPP by simplex method:
$Maximize\ z = 2x_1 + 6x_2 subject to x_1 + 10x_2 \leq 7;\ 12x_1 + 3x_2 \leq 9;$
$3x_1 + 7x_2 \leq 11;\ x_1, x_2 \geq 0.$
b) Solve the following LPP by Big-M method $Minimize z = 7x_1 + 6x_2$
$Subject to - x_1 + 9x_2 = 6;\ x_1 + 3x_2 \leq 9;\ 3x_1 - 7x_2 \geq 1, x_1, x_2 \geq 0.$

OR

2) a) Use revised simplex method to solve the following LPP
$Maximize\ z = 7x_1 + 2x_2 - 4x_3 subject\ to\ 6x_1 - 4x_2 + 5x_3 \leq 6;$
$x_1 + 2x_2 + 13x_3 = 10\ ;\ x_1, x_2, x_3 \geq 0.$
b) Explain the special cases in LP problems. During simplex procedure, how do you detect them.

UNIT-II

3) a) Explain the primal dual relationships. Write down the dual of the following LPP:
Maximize $z = 4x_1 + 3x_2 - x_3 + 3x_4$
Subject to $44x_1 + x_2 + x_3 = 12$
$-x_1 + 5x_2 + 8x_3 - 9x_4 \leq 4$
$x_1, x_2, x_4 \geq 0$; and x_3 is unrestricted in sign.

b) Use dual simplex method to obtain the optimal basic feasible solution to the LPP:
Maximize $z = -4x_1 - 6x_2 - 18x_3$
Subject to $x_1 + 3x_3 \geq 3$
$x_2 + 2x_3 \geq 5$
$x_1, x_2, x_3 \geq 0$

OR

4) Define integer linear programming problem. Solve the following linear programming problem using Branch and Bound method
Maximize $z = 4x_1 + 3x_2$
Subject to $3x_1 + x_2 \leq 15$
$3x_1 + 4x_2 \leq 24$
$x_1, x_2 \geq 0$; and are integers

UNIT-III

5) a) Explain the steps of MODI method and use it to find the optimum cost of the following transportation problem:

	T1		T2	T3	T4	Supply
A	26	14	10	12	22	
B	31	27	30	14	46	
C	15	18	16	25	12	
Demand	17	12	21	30		

b) Explain degeneracy in transportation problem and discuss the method to resolve it.

OR

6) a) Explain the mathematical formulation of assignment problem and describe the algorithm to solve the problem.

b) Solve the following assignment problem:

	I	II		III	IV	V
A	8	26	17	11	10	
B	13	28	4	26	12	
C	38	19	18	15	5	
D	19	26	24	10	8	

UNIT-IV

7) a) Use dynamic programming to solve

Maximizez $= y_1 y_2 y_3 \text{S.T. } y_1 + y_2 + y_3 = 5,\ y_1,\ y_2,\ y_3 \geq 0.$

b) Explain the main characteristic features of dynamic programming.

OR

8) Explain the procedure for solving a LP problem using dynamic programming approach.

Using Dynamin Programming, solve the following LPP

$$\text{Max } z = 3x_1 + 2x_2$$

$$\text{s.t. } x_1 + x_2 \leq 300$$

$$2x_1 + 3x_2 \leq 800$$

$$x_1, x_2 \geq 0.$$

UNIT-V

9) a) Explain Fibonacci method for solving unconstrained optimization problem.

   b) Minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ starting from the point $X_1 = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$.

OR

10) Minimize $f = x_1^2 + 3x_2^2 + 6x_3^2$ by the Hooke-Jeeve's method by taking $\Delta x_1 = \Delta x_2 = \Delta x_3 = 0.5$ and the starting point as $(2, -1, 1)$. Perform two iterations.

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M.Sc Degree Examination
Third Semester
Applied Mathematics
21AM306(C):Relativity & Cosmology-I
(Effective from the admitted batch of 2021-2022)

Time: 3 hours Maximum: 80 marks

Answer one question from each unit. All questions carry equal marks

Unit-I

1). (a) Sate and prove Quotient law of tensors.
(b) Show that the metric of a Euclidean space, referred to spherical polar coordinates $x^1 = r$, $x^2 = \theta$ and $x^3 = \psi$ is given by $ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \ d\psi^2$.

(OR)

2). (a) If $A_{ij} = B_{i,\ j} - B_{j,\ i}$, prove that $A_{i\,j,\ k} + A_{j\,k,\ i} + A_{k\,i,\ j} = 0$.
(b) If A^{ijk} is skew symmetric tensor, show that $\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^k}\left(\sqrt{g}\ A^{ijk}\right)$ is a tensor.

Unit-II

3). (a) Obtain an expression for Riemann Christoffel Tensor and also Ricci Tensor.
(b) Derive covariant curvature tensorR_{hij} and discuss properties of covariant curvature tensor.

(OR)

4). (a) Sate and prove Bianchi's identity.
(b) Prove that $R_{1212} = -G\frac{\partial^2 G}{\partial u^2}$ for the V_2 whose line- element is $ds^2 = du^2 + G^2 dv^2$, where G is a function of u and v.

Unit-III

5). (a) Show that the Lorentz transformations about to rotation of axis in space time.
(b) Explain the following terms in detail.
(i) World line. (ii) Space-like vector. (iii) Light cone.

(OR)

6). (a) Write about Minkowski-Space.
(b) Explain the transformation to proper coordinates.

Unit-IV

7). (a) Derive expression for variation of mass of a body with velocity.
(b) Derive the transformation formula for force of a body.

(OR)

8). (a) Establish the relation $E = mc^2$ and discuss the equivalence of mass and energy.
(b) Prove that$E^2 = p^2c^2 + m_0^2c^4$for all the particles in inertial frames.

Unit-V

9). (a) Show that the equation of continuity in electrodynamics in the form : $\frac{\partial \rho}{\partial t} + div\, J = 0$.
(b) Derive the transformation equations for electric and magnetic field intensities $\overline{E}$ and $\overline{H}$.

(OR)

10). Obtain Maxwell's equations in tensor form.