MODEL PAPER

B.Sc (CBCS) DEGREE EXAMINATION

Fourth Semester

Paper IV - Mathematics

REAL ANALYSIS

Time: 3 hrs Max. marks: 75

SECTION-A (5×5 = 25 marks)

Answer any FIVE of the following.

1. Test the Convergence of $\sum_{n=1}^{\infty} \frac{1}{n^3}\left[\frac{n+2}{n+3}\right]^n x^n \ \forall x>0$.

2. If $\lim_{x\to a} f(x) = l$, then show that $\lim_{x\to a} |f(x)| = l$ is the Converse true? Justify your answer.

3. Test for the Convergence of the sequence $\{S_n\}$ if
$S_n = 1+\frac{1}{2}+\frac{1}{2^2}+\cdots+\frac{1}{2^{n-1}}$

4. When do we say that the series $\sum u_n$ Converges Conditionally Show that the Series $\sum \frac{(-1)^{n-1}}{n}$ is Conditionally Convergent.

5. Examine the Continuity of the function f defined by
$f(x)=0$, if $x\in Q$; $f(x)=1$ if $x\in R-Q$.

6. Discuss the applicability of the Rolle's theorem for
$f(x)=\sqrt{1-x^2}$, $a=-1$ and $b=1$.

7. Find the θ of the Lagranges theorem for
$f(x) = x^3-2x+3$; $a=1$, $h=\frac{1}{2}$.

8. If $f(x)=x^2$ on $[0,1]$ and $P=\left\{0,\frac{1}{4},\frac{2}{4},\frac{3}{4},1\right\}$, Compute $L(P,f)$ and $U(P,f)$.

(P.T.O)

SECTION-B (5 × 10 = 50 marks)

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(a) When do we say that a sequence is bounded? Prove that every convergent sequence is bounded. Is the converse of this result true? Justify your claim.

(Or)

(b)

(i) If $S_n > 0 \ \forall n \in z^+$ and $\lim_{n\to\infty} S_n = l$, then show that $\lim (S_1, S_2, \cdots S_n)^{1/n} = l$.

ii, If $\lim S_n = l$ and $\lim t_n = l'$ then show that

$\lim (S_n t_n) = l l'$

10

(a) State and establish prove the Cauchy's n^{th} root test. Test for convergence of $\Sigma n e^{-n^2}$.

(Or)

(b) State and prove the Leibritz's test for alternating series. Hence show that $\frac{1}{1\cdot 2} - \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} - \frac{1}{7\cdot 8} + \cdots$

Converges.

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(a) If $f: [a,b] \to R$ is uniformly continuous, then prove that f is continuous in $[a,b]$. Show that the converse of this theorem need not be true.

(Or)

(b) Show that the function f defined by $f(x) = \sin \frac{1}{x}$ for every $x > 0$ is continuous but not uniformly on R^+

12.

(a) State and prove that Cauchy's mean value theorem.

(Or)

(P.T.O)

(b) Show that $f(x) = \begin{cases} x^2 \cos\frac{1}{x} & ; x \neq 0 \\ 0 & ; x = 0 \end{cases}$ is derivable every where but the derivative is not continuous at 0.

13 (a) If $f \in R[a,b]$ then prove that $|f| \in R[a,b]$. Show that the converse of this theorem is not true.

(Or)

(b) State and prove the fundamental theorem of integral calculus. Using this theorem, show that $\int_0^1 x^4 \, dx = \frac{1}{5}$.

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